Skill and Practice Worksheets
Unit 1:
1.1 Scientific Processes
1.2 Dimensional Analysis
1.2 International System of Measurements
1.2 Making Line Graphs
1.3 Speed Problems
1.3 Problem Solving Boxes *(template for solving problems)*
1.3 Working with Quantities and Rates
1.3 Problem Solving with Rates
2.1 Mass vs Weight
2.2 Acceleration Problems
2.2 Newton's Second Law
2.3 Acceleration Due to Gravity
2.4 Analyzing Graphs of Motion Without Numbers
2.4 Analyzing Graphs of Motion With Numbers
2.4 Acceleration and Speed-Time Graphs
3.1 Applying Newton’s Laws of Motion
3.1 Momentum
3.1 Impulse
3.1 Momentum Conservation
3.2 Work
3.2 Potential and Kinetic Energy
3.3 Collisions and Conservation of Momentum

Unit 2:
4.1 Work Done against Gravity
4.1 Power
4.2 Mechanical Advantage
4.2 Mechanical Advantage of Simple Machines
4.2 Gear Ratios
4.3 Efficiency
5.2 Equilibrium
5.4 Torque
6.1 Pythagorean Theorem
6.1 Adding Displacement Vectors
6.1 Projectile Motion
6.2 Circular Motion
6.3 Universal Gravitation

Unit 3:
7.1 Indirect Measurement
7.2 Temperature Scales
7.3 Specific Heat
8.1 Density
8.1 Stress
8.2 Buoyancy
8.2 Archimedes Principle
8.3 Boyle’s Law
8.3 Boyle's Law
8.3 Pressure-Temperature Relationship
8.3 Charles’ Law
9.1 The Structure of the Atom
9.2 Dot Diagrams

Unit 4:
10.2 Power in Flowing Energy
10.2 Efficiency and Energy
11.2 Balancing Chemical Equations
11.3 Radioactivity
12.1 Einstein’s Formula

Unit 5:
13.2 Using an Electric Meter
13.3 Ohm's Law
14.1 Series Circuits
14.2 Parallel Circuits
14.3 Electrical Power
15.2 Coulomb's Law

Unit 6:
16.3 Magnetic Earth
17.3 Transformers
18.1 The Inverse Square Law
18.2 Calculating Gravitational Field Strength
18.3 Calculating Electric Fields and Forces

Unit 7:
19.1 Period and Frequency
19.2 Harmonic Motion Graphs
20.1 Waves
20.1 Standing Waves
20.3 Wave Interference
21.1 Decibel Scale

Unit 8:
22.1 Light Intensity Problems
23.1 The Law of Reflection
23.2 Refraction
23.3 Ray Diagrams
24.1 The Electromagnetic Spectrum
24.1 Doppler Shift
The scientific method is a process that helps you find answers to your questions about the world. The process starts with a question and your answer to the question based on experience and knowledge. This “answer” is called your hypothesis. The next step in the process is to test your hypothesis by creating experiments that can be repeated by other people in other places. If your experiment is repeated many times with the same results and conclusions, these findings become part of the body of scientific knowledge we have about the world.

The girls decide to conduct a scientific experiment to determine whether it is faster to make ice cubes with hot water or cold water.

- Read the following story. You will use this story to practice using the scientific method.

   Maria and Elena are preparing for a party. Maria realizes she forgot to fill the ice cube trays in order to have ice for the punch. Elena says that she remembers reading somewhere that hot water freezes faster than cold water. Maria is skeptical. She learned in her physics class that the hotter the liquid, the faster the molecules are moving. Since hot water molecules have to slow down more than cold water molecules to become ice, Maria thinks that it will take hot water longer to freeze than cold water.

   The girls decide to conduct a scientific experiment to determine whether it is faster to make ice cubes with hot water or cold water.

- Now, answer the following questions about the process they used to reach their conclusion.

### Asking a question

1. What is the question that Maria and Elena want to answer by performing an experiment?

### Formulate a hypothesis

2. What is Maria’s hypothesis for the experiment? State why Maria thinks this is a good hypothesis.

### Design and conduct an experiment

3. **Variables:** There are many variables that Maria and Elena must control so that their results will be valid. Name at least four of these variables.
4. **Measurements:** List at least two types of measurements that Maria and Elena must make during their experiment.

5. **Procedure:** If Maria and Elena want their friends at the party to believe the results of their experiment, they need to conduct the experiment so that others could repeat it. Write a procedure that the girls could follow to answer their question.

**Collect and analyze data**

The girls conducted a carefully controlled experiment and found that after 3 hours and 15 minutes, the hot water had frozen solid, while the trays filled with cold water still contained a mixture of ice and water. They repeated the experiment two more times. Each time the hot water froze first. The second time they found that the hot water froze in 3 hours and 30 minutes. The third time, the hot water froze in 3 hours and 0 minutes.

6. What is the average time that it took for hot water in ice cube trays to freeze?

7. Why is it a good idea to repeat your experiments?

**Make a tentative conclusion**

8. Which of the following statements is a valid conclusion to this experiment? Explain your reasoning for choosing a certain statement.
   a. Hot water molecules don’t move faster than cold water molecules.
   b. Hot water often contains more dissolved minerals than cold water, so dissolved minerals must help water freeze faster.
   c. Cold water can hold more dissolved oxygen than hot water, so dissolved oxygen must slow down the rate at which water freezes.
   d. The temperature of water affects the rate at which it freezes.
   e. The faster the water molecules are moving, the faster they can arrange themselves into the nice, neat patterns that are found in solid ice cubes.

**Test your conclusion or refine your question**

Maria and Elena are very pleased with their experiment. They ask their teacher if they can share their findings with their science class. The teacher says that they can present their findings as long as they are sure their conclusion is correct.

Here is where the last step (step 7) is important. At the end of any set of experiments and before you present your findings, you want to make sure that you are confident about your work.

9. Let’s say that there is a small chance that the results of the experiment that Maria and Elena performed were affected by the kind of freezer they used in the experiment. What could the girls do to make sure that their results were not affected by the kind of freezer they used?

10. Conclusion 8(b) is a possible reason why temperature has an affect on how fast water freezes. Refine your original question for this experiment. In other words, create a question for an experiment that would prove or disprove conclusion (b).
Dimensional Analysis

Dimensional analysis is used to solve problems that involve converting between different units of measurement.

1. Circle the conversion factor you would choose to solve the following problems.
   a. How many inches are in 6 meters?
      \[
      \frac{1 \text{ meter}}{39.4 \text{ inches}} \quad \text{OR} \quad \frac{39.4 \text{ inches}}{1 \text{ meter}}
      \]
   b. How many liters are in 10 U.S. gallons?
      \[
      \frac{1 \text{ gallon}}{3.79 \text{ liters}} \quad \text{OR} \quad \frac{3.79 \text{ liters}}{1 \text{ gallon}}
      \]
   c. 100 kilometers is equal to how many miles?
      \[
      \frac{1 \text{ kilometer}}{0.624 \text{ miles}} \quad \text{OR} \quad \frac{0.624 \text{ miles}}{1 \text{ kilometer}}
      \]
   d. 1,000,000 grams is equal to how many kilograms?
      \[
      \frac{0.001 \text{ kilogram}}{1 \text{ gram}} \quad \text{OR} \quad \frac{1 \text{ gram}}{0.001 \text{ kilogram}}
      \]

2. A grocery store just received a shipment of 200 cartons of eggs. Each carton holds one dozen eggs. If 12 eggs = 1 dozen, how many eggs did the store receive?

3. A marathon is 26.2 miles long. How many kilometers is a marathon? (1 mile = 1.61 km)

4. The speed limit on many interstate highways in the United States is 65 miles per hour. How many kilometers per hour is that? (1 mile = 1.61 km)

5. Ashley is going on a trip to London. She has saved $100.00 in spending money. When she arrives in England, she goes to a bank to change her money into pounds. She is told that the exchange rate is 1 British pound = 1.43 American dollars. The bank charges a fee of 4 pounds to change the money from dollars to pounds. How much money, in British pounds, will Ashley have if she changes all of her dollars to pounds?

6. Although it is widely believed that Germany’s Autobahn highway has no speed limit whatsoever, much of the highway has regulated speed limits of 130 km/hr or less, and in some places speed is limited to just 60 km/hr.
   a. How many miles per hour is 130 km/hr? (1 mile = 1.61 km)
   b. How many miles per hour is 60 km/hr?

7. In England, a person’s weight is commonly given in stones. One English stone is equal to 14 pounds. If an English friend tells you he weighs eleven stones, what is his weight in pounds?
International System of Measurements

In ancient times, as trade developed between cities and nations, units of measurements were developed to measure the size of purchases and transactions. Greeks and Egyptians based their measurements of length on the human foot. Usually, it was based on the king’s foot size. The volume of baskets was measured by how much goatskin they could hold. Was this a reliable method of measurement? Why or why not?

During the Renaissance, as scientists began to develop the ideas of physics and chemistry, they needed better units of measurements to communicate scientific data more efficiently. Scientists such as Kepler, Galileo, and Newton needed to prove their ideas with data based on measurements that other scientists could reproduce.

In March 30, 1791, in Sevres, France, the French Academy of Sciences proposed a system that would be simple and consistent. The French Scientists based the units of length on a fraction of the distance between the Earth’s equator and the North Pole along a line passing through Paris. The system’s basic unit for measuring length was called a meter after the Greek word metron meaning “measure.” The set of equations below will show you how the meter is related to other units in this system of measurements.

- 1 meter = 100 centimeters
- 1 cubic centimeter = 1 cm³ = 1 milliliter
- 1000 milliliters = 1 liter

The liter was defined as the new standard for volume. One milliliter was equal to the volume of one cubic centimeter. The gram was defined as the standard for mass. The gram was defined as the mass of one milliliter of water at 4°C.

Today, the length of a meter is defined as the distance light travels in a small fraction of a second. The kilogram is the current “base unit” for mass. A kilogram is defined as the mass of a certain lump of platinum and iridium that is kept in Paris under glass to protect it from chemical changes that could alter its mass. The metric system is also called the SI system, from the French Le Systeme International d’Unites.

The United States adopted the metric system in 1884. However, the adoption process has been slow, and many Americans still use the English System (feet, inches, and pounds). Since 1992, U.S. government agencies have been required to use the metric system in business transactions.

The majority of people in the world use the metric system of measurements in their daily lives. For example, if you travel overseas or to Canada, you will find that a car’s speed is measured in kilometers per hour.
The metric system is easy to use because all the units are based on factors of 10. In the English system, there are 12 inches in a foot, 3 feet in a yard, and 1,760 yards in a mile. In the metric system, there are 10 millimeters in a centimeter, 100 centimeters in a meter, and 1,000 meters in a kilometer. From the graphic, how many kilometers is it from the North Pole to the equator? Answer: 10,000 kilometers. These are the base units of measurement that you will use in your scientific studies. The prefixes above are used with the base units when measuring very large or very small quantities.

<table>
<thead>
<tr>
<th>When you are measuring</th>
<th>Use this base unit</th>
<th>Symbol of unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>mass</td>
<td>kilogram</td>
<td>kg</td>
</tr>
<tr>
<td>length</td>
<td>meter</td>
<td>m</td>
</tr>
<tr>
<td>volume</td>
<td>liter</td>
<td>L</td>
</tr>
<tr>
<td>force</td>
<td>newton</td>
<td>N</td>
</tr>
<tr>
<td>temperature</td>
<td>degree Celsius</td>
<td>°C</td>
</tr>
<tr>
<td>time</td>
<td>second</td>
<td>sec or s</td>
</tr>
</tbody>
</table>

You may wonder why the kilogram, rather than the gram, is called the base unit for mass. This is because the mass of an object is based on how much matter it contains as compared to the standard kilogram made from platinum and iridium and kept in Paris. The gram is such a small amount of matter that if it had been used as a standard, small errors in reproducing that standard would be multiplied into very large errors when large quantities of mass were measured.

The following prefixes in the SI system indicate the multiplication factor to be used with the basic unit. For example, the prefix *kilo-* is for a factor of 1,000. A kilometer is equal to 1,000 meters and a kilogram is equal to 1,000 grams.

<table>
<thead>
<tr>
<th>Prefix</th>
<th>Symbol</th>
<th>Multiplication Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>pico–</td>
<td>p</td>
<td>0.000000000001</td>
</tr>
<tr>
<td>nano–</td>
<td>n</td>
<td>0.000000001</td>
</tr>
<tr>
<td>micro–</td>
<td>µ</td>
<td>0.000001</td>
</tr>
<tr>
<td>milli–</td>
<td>m</td>
<td>0.001</td>
</tr>
<tr>
<td>centi–</td>
<td>c</td>
<td>0.01</td>
</tr>
<tr>
<td>deci–</td>
<td>d</td>
<td>0.1</td>
</tr>
<tr>
<td>deka–</td>
<td>da</td>
<td>10</td>
</tr>
<tr>
<td>hecto–</td>
<td>h</td>
<td>100</td>
</tr>
<tr>
<td>kilo–</td>
<td>k</td>
<td>1,000</td>
</tr>
<tr>
<td>mega–</td>
<td>M</td>
<td>1,000,000</td>
</tr>
<tr>
<td>giga–</td>
<td>G</td>
<td>1,000,000,000</td>
</tr>
<tr>
<td>tera–</td>
<td>T</td>
<td>1,000,000,000,000</td>
</tr>
</tbody>
</table>
**EXAMPLES**

- **How many centimeters are in a hectometer?**
  
  Find the multiplication factor in exponent form. In this case, the multiplication factor for centi- is $10^{-2}$. The multiplication factor for hecto- is $10^2$. Now, find the absolute value of the difference of the exponents.

  $$-2 - 2 = 4$$

  The number that you get (in this case, “4”) tells you how many zeros to place after the number one (1) to get the correct answer. There are $10^4$, or 10,000 centimeters in a hectometer.

- **How many times larger is a kilogram than a gram?**

  Find the multiplication factor in exponent form. In this case, the multiplication factor for kilo- is $10^3$. A gram does not have a multiplication factor in exponent form. Find the absolute value of the difference of the exponents.

  $$3 - 0 = 3$$

  The number you get tells you how many zeros to place after the number one to get the answer. In this case, a kilogram is $10^3$ or 1,000 times larger than a gram.

**PRACTICE**

1. How many milligrams are in one gram?
2. How many centimeters are in a kilometer?
3. How many microliters are in one liter?
4. How many nanoseconds are in one second?
5. How many micrograms are in one kilogram?
6. How many milliliters are in a megaliter?
7. A deciliter is how many times larger than a milliliter?
8. A micrometer is how many times smaller than a millimeter?
9. The wavelength of red light is 650 nanometers. How much bigger is the wavelength of a water wave that measures 2 meters?
10. A first grader measures 1 meter high. How much bigger is this first grader compared to the height of a bug that measures 1 millimeter high?
11. What is the name of the distance that is $10^{-9}$ smaller than a meter?
12. What is the name of the distance that is $10^3$ larger than a meter?
13. What is the name of a volume that is 1,000 times larger than a milliliter?
14. What is the name of a mass that is $10^{-6}$ smaller than a gram?
15. A length of time that is 0.000000001 second is also called a what?
Making Line Graphs

Graphs allow you to present data in a form that is easy to understand.

1. **Data pairs:** Graphs are made using pairs of numbers. Each pair of numbers represents one data point on a graph. The first number in the pair represents the independent variable and is plotted on the $x$-axis. The second number represents the dependent variable and is plotted on the $y$-axis.

2. **Axis labels:** The label on the $x$-(horizontal) axis is the name of the independent variable. The label on the $y$- (vertical) axis is the name of the dependent variable.

3. **Data range:** The range of numbers on each axis depends on the smallest and largest value for each variable. To find the range, subtract the smallest value from the largest value for a variable.

4. **Title:** The format for the title of a graph is: “Dependent variable name versus Independent variable name.”

**PRACTICE**

1. Seven data pairs are listed in the table below. For each data pair, identify the independent and dependent variable. Then, rewrite the data pair according to the headings in the next two columns of the table. The first two data pairs are done for you.

<table>
<thead>
<tr>
<th>Data pair (not necessarily in order)</th>
<th>Independent (x-axis)</th>
<th>Dependent (y-axis)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Temperature Hours of heating</td>
<td>Hours of heating</td>
<td>Temperature</td>
</tr>
<tr>
<td>2 Stopping distance Speed of a car</td>
<td>Speed of a car</td>
<td>Stopping distance</td>
</tr>
<tr>
<td>3 Number of people in a family Cost per week for groceries</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4 Stream flow rate Amount of rainfall</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5 Tree age Average tree height</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6 Test score Number of hours studying for a test</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7 Population of a city Number of schools needed</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2. The lowest and highest values for a variable are listed in the table below. Use these values to find the data range for each variable. The first two are done for you.

<table>
<thead>
<tr>
<th>Lowest value</th>
<th>Highest value</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>28</td>
<td>28</td>
</tr>
<tr>
<td>10</td>
<td>87</td>
<td>77</td>
</tr>
<tr>
<td>0</td>
<td>4.2</td>
<td></td>
</tr>
<tr>
<td>-5</td>
<td>23</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>113</td>
<td></td>
</tr>
<tr>
<td>100</td>
<td>1250</td>
<td></td>
</tr>
</tbody>
</table>
3. Using the variable range and the number of lines, calculate the scale for an axis. The scale is the quantity represented per line on the graph. Often the calculated scale is not an easy-to-use value. To make the calculated scale easy-to-use, round the value and write this number in the column with the heading “Adjusted scale.” The first two are done for you.

<table>
<thead>
<tr>
<th>Variable range</th>
<th>Number of lines</th>
<th>Range ÷ Number of lines</th>
<th>Calculated scale</th>
<th>Adjusted scale</th>
</tr>
</thead>
<tbody>
<tr>
<td>13</td>
<td>24</td>
<td>13 ÷ 24 = 0.54</td>
<td>0.54</td>
<td>1</td>
</tr>
<tr>
<td>83</td>
<td>43</td>
<td>83 ÷ 43 = 1.93</td>
<td>1.93</td>
<td>2</td>
</tr>
<tr>
<td>31</td>
<td>35</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>100</td>
<td>33</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>300</td>
<td>20</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>900</td>
<td>15</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

4. Here is a data set for you to plot on a graph. Follow these steps to make the graph.
   a. Place this data set in the table below. Each data point is given in the format of \((x, y)\). The \(x\)-values represent time in minutes. The \(y\)-values represent distance in kilometers.
      \((0, 5.0), (10, 9.5), (20, 14.0), (30, 18.5), (40, 23.0), (50, 27.5), (60, 32.0)\).

<table>
<thead>
<tr>
<th>Independent variable (x-axis)</th>
<th>Dependent variable (y-axis)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

   b. What is the range for the independent variable?
   c. What is the range for the dependent variable?
   d. Make your graph using the blank graph below. Each axis has twenty lines (boxes). Use this information to determine the adjusted scale for the \(x\)-axis and the \(y\)-axis.
   e. Label your graph. Add a label for the \(x\)-axis, \(y\)-axis, and provide a title.
   f. Draw a smooth line through the data points.
   g. Question: What is position value after 45 minutes? Use your graph to answer this question.
   h. Question: This graph is a position versus time graph. Do you think the position change is more representative of a person running, a person on a bicycle, or a person driving a car? Justify your answer.
Speed Problems

To determine the speed of an object, you need to know the distance traveled and the time taken to travel that distance. However, by rearranging the formula for speed, \( v = \frac{d}{t} \), you can also determine the distance traveled or the time it took for the object to travel that distance, if you know the speed. For example,

<table>
<thead>
<tr>
<th>Equation...</th>
<th>Gives you...</th>
<th>If you know...</th>
</tr>
</thead>
<tbody>
<tr>
<td>( v = \frac{d}{t} )</td>
<td>speed</td>
<td>distance and time</td>
</tr>
<tr>
<td>( d = v \times t )</td>
<td>distance</td>
<td>speed and time</td>
</tr>
<tr>
<td>( t = \frac{d}{v} )</td>
<td>time</td>
<td>distance and speed</td>
</tr>
</tbody>
</table>

Use the metric system to solve the practice problems unless you are asked to write the answer using the English system of measurement. As you solve the problems, include all units and cancel appropriately.

**EXAMPLES**

**Example 1:** What is the speed of a cheetah that travels 112.0 meters in 4.0 seconds?

<table>
<thead>
<tr>
<th>Looking for</th>
<th>Speed of the cheetah.</th>
</tr>
</thead>
</table>
| Given       | Distance = 112.0 meters  
              Time = 4.0 seconds |
| Relationship| \( \text{speed} = \frac{d}{t} \) |

**Solution**

\[
\text{speed} = \frac{d}{t} = \frac{112.0 \text{ m}}{4.0 \text{ sec}} = 28 \text{ m/sec}
\]

The speed of the cheetah is 28 meters per second.

**Example 2:** There are 1,609 meters in one mile. What is this cheetah’s speed in miles/hour?

<table>
<thead>
<tr>
<th>Looking for</th>
<th>Speed of the cheetah in miles per hour.</th>
</tr>
</thead>
</table>
| Given       | Distance = 112.0 meters  
              Time = 4.0 seconds |
| Relationships| \( \text{speed} = \frac{d}{t} \)  
and 1,609 meters = 1 mile |

**Solution**

\[
\frac{28 \text{ m}}{\text{sec}} \times \frac{1 \text{ mile}}{1,609 \text{ m}} \times \frac{3,600 \text{ sec}}{1 \text{ hour}} = 63 \text{ miles/hour}
\]

The speed of the cheetah in miles per hour is 63 mph.
1. A bicyclist travels 60.0 kilometers in 3.5 hours. What is the cyclist’s average speed?

<table>
<thead>
<tr>
<th>Looking for</th>
<th>Solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>Given</td>
<td></td>
</tr>
<tr>
<td>Relationships</td>
<td></td>
</tr>
</tbody>
</table>

2. What is the average speed of a car that traveled 300.0 miles in 5.5 hours?

3. How much time would it take for the sound of thunder to travel 1,500 meters if sound travels at a speed of 330 m/sec?

4. How much time would it take for an airplane to reach its destination if it traveled at an average speed of 790 kilometers/hour for a distance of 4,700 kilometers? What is the airplane’s speed in miles/hour?

5. How far can a person run in 15 minutes if he or she runs at an average speed of 16 km/hr?
   (HINT: Remember to convert minutes to hours.)

6. In problem 5, what is the runner’s distance traveled in miles?

7. A snail can move approximately 0.30 meters per minute. How many meters can the snail cover in 15 minutes?

8. You know that there are 1,609 meters in a mile. The number of feet in a mile is 5,280. Use these equalities to answer the following problems:
   a. How many centimeters equals one inch?
   b. What is the speed of the snail in problem 7 in inches per minute?

9. Calculate the average speed (in km/h) of a car stuck in traffic that drives 12 kilometers in 2 hours.

10. How long would it take you to swim across a lake that is 900 meters across if you swim at 1.5 m/sec?
    a. What is the answer in seconds?
    b. What is the answer in minutes?

11. How far will a you travel if you run for 10 minutes at 2 m/sec?

12. You have trained all year for a marathon. In your first attempt to run a marathon, you decide that you want to complete this 26-mile race in 4.5 hours.
    a. What is the length of a marathon in kilometers (1 mile = 1.6 kilometers)?
    b. What would your average speed have to be to complete the race in 4.5 hours? Give your answer in kilometers per hour.

13. Suppose you are walking home after school. The distance from school to your home is five kilometers. On foot, you can get home in 25 minutes. However, if you rode a bicycle, you could get home in 10 minutes.
    a. What is your average speed while walking?
    b. What is your average speed while bicycling?
    c. How much faster you travel on your bicycle?
14. Suppose you ride your bicycle to the library traveling at 0.5 km/min. It takes you 25 minutes to get to the library. How far did you travel?

15. You ride your bike for a distance of 30 km. You travel at a speed of 0.75 km/minute. How many minutes does this take?

16. A train travels 225 kilometers in 2.5 hours. What is the train’s average speed?

17. An airplane travels 3,260 kilometers in 4 hours. What is the airplane’s average speed?

18. A person in a kayak paddles down river at an average speed of 10 km/h. After 3.25 hours, how far has she traveled?

19. The same person in question 18 paddles upstream at an average speed of 4 km/h. How long would it take her to get back to her starting point?

20. An airplane travels from St. Louis to Portland, Oregon in 4.33 hours. If the distance traveled is 2,742 kilometers, what is the airplane’s average speed?

21. The airplane returns to St. Louis by the same route. Because the prevailing winds push the airplane along, the return trip takes only 3.75 hours. What is the average speed for this trip?

22. The airplane refuels in St. Louis and continues on to Boston. It travels at an average speed of 610 km/h. If the trip takes 2.75 hours, what is the flight distance between St. Louis and Boston?

23. The speed of light is about $3.00 \times 10^5$ km/s. It takes approximately 1.28 seconds for light reflected from the moon to reach Earth. What is the average distance from Earth to the moon?

24. The average distance from the sun to Pluto is approximately $6.10 \times 10^9$ km. How long does it take light from the sun to reach Pluto? Use the speed of light from the previous question to help you.

25. Now, make up three speed problems of your own. Give the problems to a friend to solve and check their work.
   a. Make up a problem that involves solving for average speed.
   b. Make up a problem that involves solving for distance.
   c. Make up a problem that involves solving for time.
## Problem Solving Boxes

<table>
<thead>
<tr>
<th>Looking for</th>
<th>Solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>Given</td>
<td></td>
</tr>
<tr>
<td>Relations</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Looking for</th>
<th>Solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>Given</td>
<td></td>
</tr>
<tr>
<td>Relations</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Looking for</th>
<th>Solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>Given</td>
<td></td>
</tr>
<tr>
<td>Relations</td>
<td></td>
</tr>
</tbody>
</table>
Working with Quantities and Rates

A quantity describes an amount of something. It has two parts: a number and a unit. The number tells “how many.” The unit tells “of what.” For example, 10 apples is a quantity. 10 is the number, apples is the unit.

You cannot combine quantities unless they have the same unit. For example, 5 apples + 5 pears can’t be combined, but 5 apples + 5 apples can be combined to make 10 apples.

When you multiply or divide quantities, the units get multiplied or divided too. For example, 10 cm ÷ 10 cm = 100 cm ÷ cm, or 100 cm².

A rate describes a relationship between two quantities. Rates are commonly described as something “per” something, like “50 miles per hour.” Per means “for every” or “for each.” In science, we often use a fraction bar or slash to represent the word per, as in 10 cookies/dollar. Rates are usually written in the fraction’s lowest terms. For example, if you received $100 for working 10 hours, you could write:

\[
\frac{100 \text{ dollars}}{10 \text{ hours}} = \frac{10 \text{ dollars}}{1 \text{ hour}}
\]

Sometimes you will be asked to multiply two rates. This is often done to change one unit to another. For example, if you wanted to know how much you were paid per minute, you could set up a problem like this:

\[
\frac{10 \text{ dollars}}{1 \text{ hour}} \times \frac{1 \text{ hour}}{60 \text{ minutes}} = \frac{10 \text{ dollars}}{60 \text{ minutes}} = \frac{0.17 \text{ dollar}}{1 \text{ minute}}
\]

Notice that the rules for multiplying fractions apply to units, too. Since “hour” appears in the numerator and the denominator, the “hour” unit is cancelled.

1. Practice your skills with quantities in the problems below. Make sure that you include units in your answer. If the quantities can’t be combined, write “cannot combine” as your answer.
   a. 5 inches × 4 inches =
   b. 12 cookies – 5 cookies =
   c. 12 eggs + 12 eggs =
   d. 120 erasers ÷ 10 boxes =
   e. 12 cookies – 5 candy bars =
   f. 120 erasers ÷ 12 erasers =

   Practice your skills with rates in the problems below. Some of the units you will see are real (like seconds) and some are made up (like blinks). Even with made up units, the rules for algebra and arithmetic still apply. Make sure that you reduce fractions to their lowest terms and include units in your answer.

2. $36 =
\frac{36 \text{ dollars}}{3 \text{ hours}}

3. 48 students =
\frac{48 \text{ students}}{2 \text{ classrooms}}

4. 10 meters =
\frac{10 \text{ meters}}{\text{second}} \times \frac{60 \text{ seconds}}{1 \text{ minute}}

5. 15 winks =
\frac{15 \text{ winks}}{5 \text{ clinks}} \times \frac{10 \text{ blinks}}{5 \text{ winks}}
In the space provided, write the unit that should go in the parentheses so that each side of the equation is equal. Use the example to help you get started. Note that singular and plural units do cancel one another.

**Problem:** \( \frac{\text{miles}}{\text{( )}} \times \text{hours} = \text{miles} \)  

**Answer:** \( \frac{\text{miles}}{\text{(hour)}} \times \text{hours} = \text{miles} \)

6. \( \frac{\text{cm}}{\text{second}} \times \text{seconds} = (\quad) \)

7. \( \frac{\text{commercial}}{\text{( )}} \times \text{program} = \text{commercial} \)

8. \( \frac{\text{pound}}{\text{)} \times \text{pound} = \text{shrimp} \)

9. \( \text{seconds} \times (\quad) = \text{seconds}^2 \)

10. \( \text{cm}^2 \times (\quad) = \text{cm}^3 \)

11. \( \frac{\text{pencil}}{\text{)} \times \text{pencils} = \text{boxes} \)

12. \( \frac{\text{kg} \times \text{m}}{\text{s}^2} \times (\quad) = \text{m} \)

13. \( \text{(clink)(wink)} \times \frac{1}{\text{blinks}} = (\quad) \)

14. \( \frac{\text{miles}}{\text{hours}} \times \frac{\text{hours}}{\text{minute}} \times \frac{\text{minutes}}{\text{second}} = (\quad) \)

15. \( \frac{\text{centimeter}}{\text{hour}} \times \frac{\text{millimeter}}{\text{centimeter}} = (\quad) \)

16. \( (\quad) \times \frac{\text{pizzas}}{\text{person}} \times \frac{\text{dollars}}{\text{pizza}} = \text{dollars} \)

17. \( \frac{\text{calories}}{\text{minute}} \times \frac{\text{minute}}{\text{hour}} \times (\quad) = \text{calories} \)

18. \( \text{games} \times \frac{\$}{\text{( )}} \times \text{years} = \$ \)

19. \( \text{heartbeats} \times \frac{\text{minute}}{\text{( )}} \times \frac{\text{hour}}{\text{day}} \times \text{days} = \text{heartbeats} \)

20. \( \frac{\text{centimeters}}{\text{second}} \times \frac{\text{second}}{\text{hour}} \times \frac{\text{meter}}{\text{( )}} \times \frac{\text{meter}}{\text{meter}} \times \frac{\text{miles}}{\text{kilometer}} = \frac{\text{miles}}{\text{hour}} \)
Problem Solving with Rates

Solving mathematical problems often involves using rates. An upside down rate is called a reciprocal rate. A rate may be written as its reciprocal because no matter how you write it the rate gives you the same amount of one thing per amount of the other thing. For example, you can write 5 cookies/ $1.00 or $1.00/5 cookies. For $1.00, you know you will get 5 cookies no matter how you write the rate. In these practice problems, you will choose how you will write each rate to solve problems.

Steps for solving problems with rates are listed below. Remember, after you have set up your problem, analyze and cancel the units by crossing them out, then do the arithmetic, and provide the answer. Remember that the answer always consists of a number and a unit.

<table>
<thead>
<tr>
<th>Step</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Step 1</td>
<td>What quantity or rate are you asked for in the problem? Write it down.</td>
</tr>
<tr>
<td>Step 2</td>
<td>What do you know from reading the problem? List all known rates and quantities.</td>
</tr>
<tr>
<td>Step 3</td>
<td>Arrange the known quantities and rates to get an answer that has the right units. This arrangement might include a formula.</td>
</tr>
<tr>
<td>Step 4</td>
<td>Plug in the values you know.</td>
</tr>
<tr>
<td>Step 5</td>
<td>Solve the problem and write the answer with a number and a unit.</td>
</tr>
</tbody>
</table>

In the space provided, write the reciprocal rate of each given rate. The first one is done for you.

1. \( \frac{1 \text{ year}}{365 \text{ days}} = \frac{365 \text{ days}}{1 \text{ year}} \)

2. \( \frac{12 \text{ inches}}{\text{foot}} = \) __________

3. \( \frac{3 \text{ small pizzas}}{\$10.00} = \) __________

4. \( \frac{36 \text{ pencils}}{3 \text{ boxes}} = \) __________

5. \( \frac{18 \text{ gallons of gasoline}}{360 \text{ miles}} = \) __________
In problems 6 and 7, you will be shown how to set up steps 1-4. For step 5, you will need to solve the problem and write the answer as a number and unit.

6. Downhill skiing burns about 600 calories per hour. How many calories will you burn if you downhill ski for 3.5 hours?

   **Step 1**  Looking for calories.
   **Step 2**  600 calories/hour; 3.5 hours
   **Step 3**  \( \frac{\text{calories}}{\text{hour}} \times 3.5 \text{ hours} = \text{calorie} \)
   **Step 4**  \( \frac{600 \text{ calories}}{\text{hour}} \times 3.5 \text{ hours} = \text{calories} \)
   **Step 5**  Answer:

7. How many cans of soda will John drink in a year if he drinks 3 sodas per day? (Remember that there are 365 days in a year.)

   **Step 1**  Looking for cans of soda per year.
   **Step 2**  3 sodas/day; 365 days/year
   **Step 3**  \( \frac{\text{soda}}{\text{day}} \times \frac{\text{days}}{\text{year}} = \frac{\text{sodas}}{\text{year}} \)
   **Step 4**  \( \frac{3 \text{ sodas}}{\text{day}} \times \frac{365 \text{ days}}{\text{year}} = \frac{\text{sodas}}{\text{year}} \)
   **Step 5**  Answer:

8. How many heartbeats will a person have in a week if he has an average heart rate of 72 beats per minute? (Remember the days/week, hours/day, and minutes/hour.

   **Step 1**  Looking for number of heartbeats per week.
   **Step 2**  72 heartbeats/minute, 7 days/week, 24 hours/day, 60 minutes/hour
   **Step 3**  \( \frac{\text{heartbeats}}{\text{minute}} \times \frac{\text{minutes}}{\text{hour}} \times \frac{\text{hours}}{\text{day}} \times \frac{\text{days}}{\text{week}} = \frac{\text{heartbeats}}{\text{week}} \)
   **Step 4**  \( \frac{72 \text{ heartbeats}}{\text{minute}} \times \frac{60 \text{ minutes}}{\text{hour}} \times \frac{24 \text{ hours}}{\text{day}} \times \frac{7 \text{ days}}{\text{week}} = \frac{\text{heartbeats}}{\text{week}} \)
   **Step 5**  Answer:
Using the five problem-solving steps, solve the following problems on your own. Be sure to read the problem carefully. Show your work in the blank provided.

<table>
<thead>
<tr>
<th>Step</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>What quantity or rate are you asked for in the problem? Write it down.</td>
</tr>
<tr>
<td>2.</td>
<td>What do you know from reading the problem? List all known rates and quantities.</td>
</tr>
<tr>
<td>3.</td>
<td>Arrange the known quantities and rates to get an answer that has the right units. This arrangement might include a formula.</td>
</tr>
<tr>
<td>4.</td>
<td>Plug in the values you know.</td>
</tr>
<tr>
<td>5.</td>
<td>Solve the problem and write the answer with a number and a unit.</td>
</tr>
</tbody>
</table>

9. How much will you pay for 5 pounds of shrimp if the cost is 2 pounds for $10.99?

10. How many miles can you get on one tank of gas if your tank holds 18 gallons and you get 22 miles per gallon?

11. What is your rate in miles/hour if you run at a speed of 2.2 miles in 20 minutes?

12. Suppose for your cookout you need to make 100 hamburgers. You know that 2.00 pounds will make 9 hamburgers. How many pounds will you need?

13. What is your mass in kilograms if you weigh 120 pounds? (There are approximately 2.2 pounds in one kilogram.)

14. Mt. Everest is 29,028 feet high. How many miles is this? (There are 5,280 feet in one mile.)

15. Susan works 8 hours a day and makes $7.00 per hour. How much money does Susan earn in one week if she works 5 days per week?

16. How many years will it take a major hamburger fast food chain to sell 45,000,000 burgers if it sells approximately 12,350 burgers per day?

17. Your science teacher needs to make more of a salt-water mixture. The concentration of the mixture that is needed is 35 grams of salt in 1,000 milliliters of water. How many grams of salt will be needed to make 1,500 milliliters of the salt-water?

18. A cart travels down a ramp at an average speed of 5.00 centimeters/second. What is the speed of the cart in miles/hour? (Remember there are 100 centimeters per meter, 1000 meters/kilometer, and 1.6 kilometer per mile.)

19. A person goes to the doctor and will need a 3-month prescription of medicine. The person will be required to take 3 pills per day. How many pills will the doctor write the prescription for assuming there are 30 days in a month?

20. If you are traveling at 65 miles per hour, how many feet will you be traveling in one second?
Mass vs Weight

What is the difference between mass and weight?

<table>
<thead>
<tr>
<th>mass</th>
<th>weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Mass is a measure of the amount of matter in an object. Mass is not related to gravity.</td>
<td>• Weight is a measure of the gravitational force between two objects.</td>
</tr>
<tr>
<td>• The mass of an object does not change when it is moved from one place to another.</td>
<td>• The weight of an object does change when the amount of gravitational force changes, as when an object is moved from Earth to the moon.</td>
</tr>
<tr>
<td>• Mass is commonly measured in grams or kilograms.</td>
<td>• Weight is commonly measured in newtons or pounds.</td>
</tr>
</tbody>
</table>

Weightlessness: When a diver dives off of a 10-meter diving board, she is in free-fall. If the diver jumped off of the board with a scale attached to her feet, the scale would read zero even though she is under the influence of gravity. She is “weightless” because her feet have nothing to push against. Similarly, astronauts and everything inside a space shuttle seem to be weightless because they are in constant free fall. The space shuttle moves at high speed, therefore, its constant fall toward Earth results in an orbit around the planet.

EXAMPLES

• On Earth’s surface, the force of gravity acting on one kilogram is 2.22 pounds. So, if an object has a mass of 3.63 kilograms, the force of gravity acting on that mass on Earth will be:

$$3.63 \text{ kg} \times \frac{2.22 \text{ pounds}}{\text{kg}} = 8.06 \text{ pounds}$$

• On the moon’s surface, the force of gravity is about 0.370 pounds per kilogram. The same newborn baby, if she traveled to the moon, would have a mass of 3.63 kilograms, but her weight would be just 1.33 pounds.

$$3.63 \text{ kg} \times \frac{0.370 \text{ pounds}}{\text{kg}} = 1.33 \text{ pounds}$$

PRACTICE

1. What is the weight (in pounds) of a 7.0-kilogram bowling ball on Earth’s surface?
2. What is the weight of a 7.0-kilogram bowling ball on the surface of the moon?
3. What is the mass of a 7.0-kilogram bowling ball on the surface of the moon?
4. Describe what would happen to the spring in a bathroom scale if you were on the moon when you stepped on it. How is this different from stepping on the scale on Earth?
5. Would a balance function correctly on the moon? Why or why not?
6. Activity: Take a bathroom scale into an elevator. Step on the scale.
   a. What happens to the reading on the scale as the elevator begins to move upward? to move downward?
   b. What happens to the reading on the scale when the elevator stops moving?
   c. Why does your weight appear to change, even though you never left Earth’s gravity?
Acceleration Problems

- Acceleration is the rate of change in the speed of an object. To determine the rate of acceleration, you use the formula below. The units for acceleration are meters per second per second or m/sec².

\[
\text{Acceleration} = \frac{\text{Final speed} - \text{Beginning speed}}{\text{Time}}
\]

\[a = \frac{v_2 - v_1}{t}\]

- A positive value for acceleration refers to the rate of speeding up, and negative value for acceleration refers to the rate of slowing down. The rate of slowing down is also called deceleration.

- The acceleration formula can be rearranged to solve for other variables such as final speed \((v_2)\) and time \((t)\).

\[v_2 = v_1 + (a \times t)\]

\[t = \frac{v_2 - v_1}{a}\]

**EXAMPLES**

- A skater increases her velocity from 2.0 m/sec to 10.0 m/sec in 3.0 seconds. What is the skater’s acceleration?

<table>
<thead>
<tr>
<th>Looking for</th>
<th>Solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>Acceleration of the skater</td>
<td>Acceleration = (\frac{10.0 \text{ m}}{3 \text{ sec}} - \frac{2.0 \text{ m}}{3 \text{ sec}}) = (\frac{8.0 \text{ m}}{3 \text{ sec}}) = (\frac{2.7 \text{ m}}{\text{sec}^2})</td>
</tr>
<tr>
<td>Given</td>
<td>The acceleration of the skater is 2.7 meters per second per second.</td>
</tr>
<tr>
<td>Beginning speed = 2.0 m/sec</td>
<td></td>
</tr>
<tr>
<td>Final speed = 10.0 m/sec</td>
<td></td>
</tr>
<tr>
<td>Change in time = 3 seconds</td>
<td></td>
</tr>
</tbody>
</table>

\[a = \frac{v_2 - v_1}{t}\]
2.2

A car accelerates at a rate of 3.0 m/sec\(^2\). If its original speed is 8.0 m/sec, how many seconds will it take the car to reach a final speed of 25.0 m/sec?

<table>
<thead>
<tr>
<th>Looking for</th>
<th>Solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>The time to reach the final speed.</td>
<td>Time (= \frac{25.0 \text{ m/sec} - 8.0 \text{ m/sec}}{3.0 \text{ m/sec}^2}) = 5.7 sec</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Given</th>
<th>The time for the car to reach its is 5.7 seconds.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beginning speed = 8.0 m/sec</td>
<td>Time (= \frac{17.0 \text{ m/sec}}{3.0 \text{ m/sec}^2}) = 5.7 sec</td>
</tr>
<tr>
<td>Final speed = 25.0 m/sec</td>
<td></td>
</tr>
<tr>
<td>Acceleration = 3.0 m/sec(^2)</td>
<td></td>
</tr>
</tbody>
</table>

\[
t = \frac{v_2 - v_1}{a}
\]

1. While traveling along a highway a driver slows from 24 m/sec to 15 m/sec in 12 seconds. What is the automobile’s acceleration? (Remember that a negative value indicates a slowing down or deceleration.)

<table>
<thead>
<tr>
<th>Looking for</th>
<th>Solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>The time to reach the final speed.</td>
<td>Time (= \frac{17.0 \text{ m/sec}}{3.0 \text{ m/sec}^2}) = 5.7 sec</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Given</th>
<th>The time for the car to reach its is 5.7 seconds.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beginning speed = 24 m/sec</td>
<td>Time (= \frac{17.0 \text{ m/sec}}{3.0 \text{ m/sec}^2}) = 5.7 sec</td>
</tr>
<tr>
<td>Final speed = 15 m/sec</td>
<td></td>
</tr>
</tbody>
</table>

2. A parachute on a racing dragster opens and changes the speed of the car from 85 m/sec to 45 m/sec in a period of 4.5 seconds. What is the acceleration of the dragster?

3. The cheetah, which is the fastest land mammal, can accelerate from 0.0 mi/hr to 70.0 mi/hr in 3.0 seconds. What is the acceleration of the cheetah? Give your answer in units of mph/sec.

4. The Lamborghini Diablo sports car can accelerate from 0.0 km/hr to 99.2 km/hr in 4.0 seconds. What is the acceleration of this car? Give your answer in units of kilometers per hour/sec.

5. Which has greater acceleration, the cheetah or the Lamborghini Diablo? (To figure this out, you must remember that there are 1.6 kilometers in 1 mile.) Be sure to show your calculations.
6. The table below includes data for a ball rolling down a hill. Fill in the missing data values in the table and determine the acceleration of the rolling ball.

<table>
<thead>
<tr>
<th>Time (seconds)</th>
<th>Speed (km/h)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 (start)</td>
<td>0 (start)</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>6</td>
</tr>
<tr>
<td>8</td>
<td>9</td>
</tr>
<tr>
<td>10</td>
<td>15</td>
</tr>
</tbody>
</table>

7. A car traveling at a speed of 30.0 m/sec encounters an emergency and comes to a complete stop. How much time will it take for the car to stop if its rate of deceleration is -4.0 m/sec\(^2\)?

8. If a car can go from 0.0 to 60.0 mi/hr in 8.0 seconds, what would be its final speed after 5.0 seconds if its starting speed were 50.0 mi/hr?

9. A cart rolling down an incline for 5.0 seconds has an acceleration of 4.0 m/sec\(^2\). If the cart has a beginning speed of 2.0 m/sec, what is its final speed?

10. A helicopter’s speed increases from 25 m/sec to 60 m/sec in 5 seconds. What is the acceleration of this helicopter?

11. Below are three situations. In which is the acceleration most similar to the helicopters in question 10?
   a. An object going from 0 m/sec to 25 m/sec in 5 seconds.
   b. An object going from 0 m/sec to 7 m/sec in 2 seconds.
   c. An object going from 5 km/h to 30 km/h in 1 second.
Newton's Second Law

- Newton’s second law states that the acceleration of an object is directly related to the force on it, and inversely related to the mass of the object. You need more force to move or stop an object with a lot of mass (or inertia) than you need for an object with less mass.
- The formula for the second law of motion (first row below) can be rearranged to solve for mass and force.

What do you want to know? | What do you know? | The formula you will use
--- | --- | ---
acceleration \((a)\) | force \((F)\) and mass \((m)\) | acceleration \(=\) force \(\div\) mass
mass \((m)\) | acceleration \((a)\) and force \((F)\) | mass \(=\) force \(\div\) acceleration
force \((F)\) | acceleration \((a)\) and mass \((m)\) | force \(=\) acceleration \(\times\) mass

**EXAMPLES**

- How much force is needed to accelerate a truck with a mass of 2,000 kilograms at a rate of 3\(\text{m/sec}^2\)?

\[
F = m \times a = 2,000 \text{ kg} \times \frac{3 \text{ m}}{\text{sec}^2} = 6,000 \text{ kg} \cdot \frac{m}{\text{sec}^2} = 6,000 \text{ N}
\]

- What is the mass of an object that requires 15 \(\text{N}\) to accelerate it at a rate of 1.5 \(\text{m/sec}^2\)?

\[
m = \frac{F}{a} = \frac{15 \text{ kg-m}}{\frac{15 \text{ m}}{1.5 \text{ m}}} = \frac{15 \text{ kg-m}}{1.5 \text{ sec}^2} = 10 \text{ kg}
\]

**PRACTICE**

1. What is the rate of acceleration of a 2,000-kilogram truck if a force of 4,200 \(\text{N}\) is used to make it start moving forward?

2. What is the acceleration of a 0.30 kilogram ball that is hit with a force of 25 \(\text{N}\)?

3. How much force is needed to accelerate a 68 kilogram-skier at a rate of 1.2 \(\text{m/sec}^2\)?

4. What is the mass of an object that requires a force of 30 \(\text{N}\) to accelerate at a rate of 5 \(\text{m/sec}^2\)?

5. What is the force on a 1,000 kilogram-elevator that is falling freely under the acceleration of gravity only?

6. What is the mass of an object that needs a force of 4,500 \(\text{N}\) to accelerate it at a rate of 5 \(\text{m/sec}^2\)?

7. What is the acceleration of a 6.4 kilogram bowling ball if a force of 12 \(\text{N}\) is applied to it?
Acceleration due to gravity is known to be 9.8 meters/second/second or 9.8 m/sec² and is represented by \( g \). Three conditions must be met before we can use this acceleration: (1) the object must be in free fall, (2) the object must have negligible air resistance, and (3) the object must be close to the surface of Earth.

In all of the examples and problems, we will assume that these conditions have been met. Remember that speed refers to “how fast” in any direction, but velocity refers to “how fast” in a specific direction. The sign of numbers in these calculations is important. Velocities upward shall be positive, and velocities downward shall be negative. Because the \( y \)-axis of a graph is vertical, change in height shall be indicated by \( y \).

Here is the equation for solving for velocity:

\[
\text{final velocity} = \text{initial velocity} + (\text{the acceleration due to the force of gravity} \times \text{time})
\]

OR

\[
v = v_0 + gt
\]

Imagine that an object falls for one second. We know that at the end of the second it will be traveling at 9.8 meters/second. However, it began its fall at zero meters/second. Therefore, its average velocity is half of 9.8 meters/second. We can find distance by multiplying this average velocity by time. Here is the equation for solving for distance. Look to find these concepts in the equation:

\[
\text{distance} = \frac{\text{the acceleration due to the force of gravity} \times \text{time}}{2} \times \text{time}
\]

OR

\[
y = \frac{1}{2}gt^2
\]

**Examples**

**Example 1:** How fast will a pebble be traveling 3 seconds after being dropped?

\[
v = v_0 + gt
\]

\[
v = 0 + (-9.8 \text{ meters/sec}^2 \times 3 \text{ sec})
\]

\[
v = -29.4 \text{ meters/sec}
\]

(Note that \( gt \) is negative because the direction is downward.)
Example 2: A pebble dropped from a bridge strikes the water in exactly 4 seconds. How high is the bridge?

\[ y = \frac{1}{2}gt^2 \]

\[ y = \frac{1}{2} \times 9.8 \text{ meters/sec} \times 4 \text{ sec} \times 4 \text{ sec} \]

\[ y = \frac{1}{2} \times 9.8 \text{ meters/sec}^2 \times 4 \text{ sec} \times 4 \text{ sec} \]

\[ y = 78.4 \text{ meters} \]

Note that the terms cancel. The answer written with the correct number of significant figures is 78 meters. The bridge is 78 meters high.

**PRACTICE**

1. A penny dropped into a wishing well reaches the bottom in 1.50 seconds. What was the velocity at impact?
2. A pitcher threw a baseball straight up at 35.8 meters per second. What was the ball’s velocity after 2.50 seconds? (Note that, although the baseball is still climbing, gravity is accelerating it downward.)
3. In a bizarre but harmless accident, Superman fell from the top of the Eiffel Tower. How fast was Superman traveling when he hit the ground 7.80 seconds after falling?
4. A water balloon was dropped from a high window and struck its target 1.1 seconds later. If the balloon left the person’s hand at –5.0 m/sec, what was its velocity on impact?
5. A stone tumbles into a mine shaft and strikes bottom after falling for 4.2 seconds. How deep is the mine shaft?
6. A boy threw a small bundle toward his girlfriend on a balcony 10.0 meters above him. The bundle stopped rising in 1.5 seconds. How high did the bundle travel? Was that high enough for her to catch it?
7. A volleyball serve was in the air for 2.2 seconds before it landed untouched in the far corner of the opponent’s court. What was the maximum height of the serve?

The equations demonstrated so far can be used to find time of flight from speed or distance, respectively. Remember that an object thrown into the air represents two mirror-image flights, one up and the other down.

<table>
<thead>
<tr>
<th>Time from velocity</th>
<th>Original equation</th>
<th>Rearranged equation to solve for time</th>
</tr>
</thead>
<tbody>
<tr>
<td>( v = v_0 + gt )</td>
<td>( t = \frac{v - v_0}{g} )</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Time from distance</th>
<th>Original equation</th>
<th>Rearranged equation to solve for time</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y = \frac{1}{2}gt^2 )</td>
<td>( t = \sqrt{\frac{2y}{g}} )</td>
<td></td>
</tr>
</tbody>
</table>

8. At about 55 meters/sec, a falling parachuter (before the parachute opens) no longer accelerates. Air friction opposes acceleration. Although the effect of air friction begins gradually, imagine that the parachuter is free falling until terminal speed (the constant falling speed) is reached. How long would that take?
9. The climber dropped her compass at the end of her 240-meter climb. How long did it take to strike bottom?
10. For practice and to check your understanding, use these equations to check your work in Sections 2 and 3.
Analyzing Graphs of Motion Without Numbers

Position-time graphs

The graph at right represents the story of “The Three Little Pigs.” The parts of the story are listed below.

- The wolf started from his house. The graph starts at the origin.
- Traveled to the straw house. The line moves upward.
- Stayed to blow it down and eat dinner. The line is flat because position is not changing.
- Traveled to the stick house. The line moves upward again.
- Again stayed, blew it down, and ate seconds. The line is flat.
- Traveled to the brick house. The line moves upward.
- Died in the stew pot at the brick house. The line is flat.

The graph illustrates that the pigs’ houses are generally in a line away from the wolf’s house and that the brick house was the farthest away.

Speed-time graphs

A speed-time graph displays the speed of an object over time and is based on position-time data. Speed is the relationship between distance (position) and time, \( v = \frac{d}{t} \). For the first part of the wolf’s trip in the position versus time graph, the line rises steadily. This means the speed for this first leg is constant. If the wolf traveled this first leg faster, the slope of the line would be steeper.

The wolf moved at the same speed toward his first two “visits.” His third trip was slightly slower. Except for this slight difference, the wolf was either at one speed or stopped (shown by a flat line in the speed versus time graph).

Read the steps for each story. Sketch a position-time graph and a speed-time graph for each story.

1. Graph Red Riding Hood's movements according the following events listed in the order they occurred:

- Little Red Riding Hood set out for Grandmother’s cottage at a good walking pace.
- She stopped briefly to talk to the wolf.
- She walked a bit slower because they were talking as they walked to the wild flowers.
- She stopped to pick flowers for quite a while.
- Realizing she was late, Red Riding Hood ran the rest of the way to Grandmother’s cottage.
2. Graph the movements of the Tortoise and the Hare. Use two lines to show the movements of each animal on each graph. The movements of each animal are listed in the order they occurred.

- The tortoise and the hare began their race from the combined start-finish line. By the end of the race, the two will be at the same position at which they started.
- Quickly outdistancing the tortoise, the hare ran off at a moderate speed.
- The tortoise took off at a slow but steady speed.
- The hare, with an enormous lead, stopped for a short nap.
- With a start, the hare awoke and realized that he had been sleeping for a long time.
- The hare raced off toward the finish at top speed.
- Before the hare could catch up, the tortoise’s steady pace won the race with an hour to spare.

3. Graph the altitude of the sky rocket on its flight according to the following sequence of events listed in order.

- The skyrocket was placed on the launcher.
- As the rocket motor burned, the rocket flew faster and faster into the sky.
- The motor burned out; although the rocket began to slow, it continued to coast ever higher.
- Eventually, the rocket stopped for a split second before it began to fall back to Earth.
- Gravity pulled the rocket faster and faster toward Earth until a parachute popped out, slowing its descent.
- The descent ended as the rocket landed gently on the ground.

4. A story told from a graph: Tim, a student at Cumberland Junior High, was determined to ask Caroline for a movie date. Use these graphs of his movements from his house to Caroline’s to write the story.
Analyzing Graphs of Motion With Numbers

Speed can be calculated from position-time graphs and distance can be calculated from speed-time graphs. Both calculations rely on the familiar speed equation: \( v = \frac{d}{t} \).

This graph shows position and time for a sailboat starting from its home port as it sailed to a distant island. By studying the line, you can see that the sailboat traveled 10 miles in 2 hours.

**EXAMPLES**

- **Calculating speed from a position-time graph**

  The speed equation allows us to calculate that the vessel speed during this time was 5 miles per hour.

  \[
  v = \frac{d}{t} \\
  v = \frac{10 \text{ miles}}{2 \text{ hours}} \\
  v = 5 \text{ miles/hour}, \text{ read as 5 miles per hour}
  \]

  This result can now be transferred to a speed-time graph. Remember that this speed was measured during the first two hours.

  The line showing vessel speed is horizontal because the speed was constant during the two-hour period.

- **Calculating distance from a speed-time graph**

  Here is the speed-time graph of the same sailboat later in the voyage. Between the second and third hours, the wind freshened and the sailboat increased its speed to 7 miles per hour. The speed remained 7 miles per hour to the end of the voyage.

  How far did the sailboat go during this time? We will first calculate the distance traveled between the third and sixth hours.
On a speed-time graph, distance is equal to the area between the baseline and the plotted line. You know that the area of a rectangle is found with the equation: $A = L \times W$. Similarly, multiplying the speed from the $y$-axis by the time on the $x$-axis produces distance. Notice how the labels cancel to produce miles:

$$\text{speed} \times \text{time} = \text{distance}$$

$$7 \text{ miles/hour} \times (6 \text{ hours} - 3 \text{ hours}) = \text{distance}$$

$$7 \text{ miles/hour} \times 3 \text{ hours} = \text{distance} = 21 \text{ miles}$$

Now that we have seen how distance is calculated, we can consider the distance covered between hours 2 and 3.

The easiest way to visualize this problem is to think in geometric terms. Find the area of the rectangle labeled “1st problem,” then find the area of the triangle above, and add the two areas.

---

**Area of triangle A**

**Geometry formula**

$$\text{speed} \times \text{time} = \text{distance}$$

$$\left(7 \text{ miles/hour} - 5 \text{ miles/hour}\right) \times \frac{(3 \text{ hours} - 2 \text{ hours})}{2} = \text{distance} = 1 \text{ mile}$$

**Area of rectangle B**

**Geometry formula**

$$\text{speed} \times \text{time} = \text{distance}$$

$$5 \text{ miles/hour} \times (3 \text{ hours} - 2 \text{ hours}) = \text{distance} = 5 \text{ miles}$$

**Add the two areas**

$$\text{Area A} + \text{Area B} = \text{distance}$$

$$1 \text{ mile} + 5 \text{ mile} = \text{distance} = 6 \text{ miles}$$

---

We can now take the distances found for both sections of the speed graph to complete our position-time graph:
1. For each position-time graph, calculate and plot speed on the speed-time graph to the right.
   a. The bicycle trip through hilly country
   ![Position vs. Time](image1) ![Speed vs. Time](image2)
   b. A walk in the park
   ![Position vs. Time](image3) ![Speed vs. Time](image4)
   c. Strolling up and down the supermarket aisles
   ![Position vs. Time](image5) ![Speed vs. Time](image6)
2. For each speed-time graph, calculate and plot the distance on the position-time graph to the right. For this practice, assume that movement is always away from the starting position.

a. The honey bee among the flowers

b. Rover runs the street

c. The amoeba
Acceleration and Speed-Time Graphs

Acceleration is the rate of change in the speed of an object. The graph below shows that object A accelerated from rest to 10 miles per hour in two hours. The graph also shows that object B took four hours to accelerate from rest to the same speed. Therefore, object A accelerated twice as fast as object B.

Calculating acceleration from a speed-time graph

The steepness of the line in a speed-time graph is related to acceleration. This angle is the slope of the line and is found by dividing the change in the $y$-axis value by the change in the $x$-axis value.

\[
\text{Acceleration} = \frac{\Delta y}{\Delta x}
\]

In everyday terms, we can say that the speed of object A "increased 10 miles per hour in two hours." Using the slope formula:

\[
\text{Acceleration} = \frac{\Delta y}{\Delta x} = \frac{10 \text{ mph} - 0 \text{ mph}}{2 \text{ hours} - 0 \text{ hours}} = \frac{5 \text{ mph}}{\text{hour}}
\]

- Acceleration = $\Delta y/\Delta x$ (the symbol $\Delta$ means “change in”)
- Acceleration = $(10 \text{ mph} - 0 \text{ mph})/(2 \text{ hours} - 0 \text{ hours})$
- Acceleration = $5 \text{ mph/hour}$ (read as 5 miles per hour per hour)

Beginning physics students are often thrown by the double *per time* label attached to all accelerations. It is not so alien a concept if you break it down into its parts:

\[
\begin{array}{c|c}
\text{The speed changes. . .} & \ldots \text{during this amount of time:} \\
\hline
5 \text{ miles per hour} & \text{per hour}
\end{array}
\]

Accelarations can be negative. If the line slopes downward, $\Delta y$ will be a negative number because a larger value of $y$ will be subtracted from a smaller value of $y$.

Calculating distance from a speed-time graph

The area between the line on a speed-time graph and the baseline is equal to the distance that an object travels. This follows from the rate formula:

\[
\text{Rate or Speed} = \frac{\text{Distance}}{\text{Time}}
\]

\[
v = \frac{d}{t}
\]

Or, rewritten:

\[
v \times t = d
\]

\[
\text{miles/hour} \times 3 \text{ hours} = 3 \text{ miles}
\]
Notice how the labels cancel to produce a new label that fits the result.

Here is a speed-time graph of a boat starting from one place and sailing to another:

The graph shows that the sailboat accelerated between the second and third hour. We can find the total distance by finding the area between the line and the baseline. The easiest way to do that is to break the area into sections that are easy to solve and then add them together.

\[ A + B + C + D = \text{distance} \]

- Use the formula for the area of a rectangle, \( A = L \times W \), to find areas A, B, and D.
- Use the formula for finding the area of a triangle, \( A = l \times w/2 \), to find area C.

\[ A + B + C + D = \text{distance} \]

10 miles + 5 miles + 1 mile + 21 miles = 37 miles

**PRACTICE**

Calculate acceleration from each of these graphs.

1. Graph 1:

2. Graph 2:
3. Graph 3:

4. Find acceleration for segment 1 and segment 2 in this graph:

5. Calculate total distance for this graph:

6. Calculate total distance for this graph:

7. Calculate total distance for this graph:
Applying Newton’s Laws of Motion

In the second column of the table below, write each of Newton’s three laws of motion. Use your own wording. In the third column of the table, describe an example of each law. To find examples of Newton’s laws, think about all the activities you do in one day.

<table>
<thead>
<tr>
<th>Newton’s laws of motion</th>
<th>Write the law here in your own words</th>
<th>Example of the law</th>
</tr>
</thead>
<tbody>
<tr>
<td>The first law</td>
<td></td>
<td></td>
</tr>
<tr>
<td>The second law</td>
<td></td>
<td></td>
</tr>
<tr>
<td>The third law</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

1. When Jane drives to work, she always places her purse on the passenger’s seat. By the time she gets to work, her purse has fallen on the floor in front of the passenger seat. One day, she asks you to explain why this happens in terms of physics. What do you say?

2. You are waiting in line to use the diving board at your local pool. While watching people dive into the pool from the board, you realize that using a diving board to spring into the air before a dive is a good example of Newton’s third law of motion. Explain how a diving board illustrates Newton’s third law of motion.

3. You know the mass of an object and the force applied to the object to make it move. Which of Newton’s laws of motion will help you calculate the acceleration of the object?

4. How many newtons of force are represented by the following amount: 3 kg·m/sec²? Select the correct answer (a, b, or c) and justify your answer.
   a. 6 newtons
   b. 3 newtons
   c. 1 newton

5. Your shopping cart has a mass of 65 kilograms. In order to accelerate the shopping cart down an aisle at 0.3 m/sec², what force would you need to use or apply to the cart?

6. A small child has a wagon with a mass of 10 kilograms. The child pulls on the wagon with a force of 2 newtons. What is the acceleration of the wagon?

7. You dribble a basketball while walking on a basketball court. List and describe the pairs of action-reaction forces in this situation.
Momentum

Which is more difficult to stop: A tractor-trailer truck barreling down the highway at 35 meters per second, or a small two-seater sports car traveling the same speed?

You probably guessed that it takes more force to stop a large truck than a small car. In physics terms, we say that the truck has greater momentum.

We can find momentum using this equation:

\[
m = \text{mass of object} \times \text{velocity of object}
\]

Velocity is a term that refers to both speed and direction. For our purposes we will assume that the vehicles are traveling in a straight line. In that case, velocity and speed are the same.

The equation for momentum is abbreviated like this: \( p = m \times v \).

Momentum, symbolized with a \( p \), is expressed in units of kg·m/sec; \( m \) is the mass of the object, in kilograms; and \( v \) is the velocity of the object in m/sec.

Use your knowledge about solving equations to work out the following problems:

1. If the truck has a mass of 2,000 kilograms, what is its momentum? Express your answer in kg·m/sec.

2. If the car has a mass of 1,000 kilograms, what is its momentum?

3. An 8-kilogram bowling ball is rolling in a straight line toward you. If its momentum is 16 kg·m/sec, how fast is it traveling?

4. A beach ball is rolling in a straight line toward you at a speed of 0.5 m/sec. Its momentum is 0.25 kg·m/sec. What is the mass of the beach ball?

5. A 4,000-kilogram truck travels in a straight line at 10.0 m/sec. What is its momentum?

6. A 1,400-kilogram car is also traveling in a straight line. Its momentum is equal to that of the truck in the previous question. What is the velocity of the car?

7. Which would take more force to stop in 10 seconds: an 8.0-kilogram ball rolling in a straight line at a speed of 0.2 m/sec or a 4.0-kilogram ball rolling along the same path at a speed of 1.0 m/sec?

8. The momentum of a car traveling in a straight line at 20 m/sec is 24,500 kg·m/sec. What is the car’s mass?

9. A 0.14-kilogram baseball is thrown in a straight line at a velocity of 30 m/sec. What is the momentum of the baseball?

10. Another pitcher throws the same baseball in a straight line. Its momentum is 2.1 kg·m/sec. What is the velocity of the ball?

11. A 1-kilogram turtle crawls in a straight line at a speed of 0.01 m/sec. What is the turtle’s momentum?
**Impulse**

A change in momentum for an object is equal to impulse. Momentum changes when velocity changes.

\[
\text{impulse} = \text{change in momentum}
\]

Force is what changes velocity. Therefore, when momentum changes a force must be involved for a period of time. The following equation relates impulse to change in momentum.

\[
F \times t = m v_2 - m v_1
\]

Momentum \((p)\) is expressed in units of kg·m/s; \(m\) is the mass of the object, in kg; and \(v\) is the velocity of the object in m/sec. Impulse is expressed in units of N·sec.

1 N·sec = 1 kg·m/sec because 1 newton = 1 kg·m/sec²:

\[
1 \text{ N·sec} = 1 \frac{\text{kg·m}}{\text{sec}^2} \times \text{sec} = 1 \frac{\text{kg·m}}{\text{sec}}
\]

**EXAMPLES**

A net force of 50 newtons is applied to a 20-kilogram cart that is already moving at 1 meter per second. The final speed of the cart was 3 meters per second. For how long was the force applied?

<table>
<thead>
<tr>
<th>Looking for</th>
<th>The speed of the cart after 3 seconds.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Given</td>
<td>Force applied = 50 newtons</td>
</tr>
<tr>
<td></td>
<td>Mass of the car = 20 kilograms</td>
</tr>
<tr>
<td></td>
<td>Initial speed of the cart = 1 m/sec</td>
</tr>
<tr>
<td></td>
<td>Final speed of the cart = 3 m/sec</td>
</tr>
<tr>
<td>Relationships</td>
<td>( t = \frac{mv_2 - mv_1}{F} )</td>
</tr>
</tbody>
</table>

\[
t = \frac{(20 \text{ kg})(3 \text{ m/sec}) - (20 \text{ kg})(1 \text{ m/sec})}{50 \text{ N}}
\]

\[
t = \frac{(60 \text{ kg·m/sec}) - (20 \text{ kg·m/sec})}{50 \text{ N}}
\]

\[
t = \frac{(40 \text{ kg·m/sec})}{50 \text{ N}} = 0.8 \text{ sec}
\]

The force was applied to the cart for 0.8 second.
1. A net force of 100 newtons is applied to a 20-kilogram cart that is already moving at 3 meter per second. The final speed of the cart was 8 meters per second. For how long was the force applied?

<table>
<thead>
<tr>
<th>Looking for</th>
<th>Solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>Given</td>
<td></td>
</tr>
<tr>
<td>Relationships</td>
<td></td>
</tr>
</tbody>
</table>

2. A 3-kilogram ball is accelerated from rest to a speed of 10 m/sec.
   a. What is the ball’s change in momentum?
   b. What is the impulse?
   c. If a constant force of 40 newtons is applied to change the momentum in this situation, for how long does the force act?

3. A 2,000-kilogram car uses a braking force of 12,000 newtons to stop in 5 seconds.
   a. What impulse acts on the car?
   b. What is the change in momentum of the car?
   c. What is the initial speed of the car?

4. A 60-kilogram high jumper lands on a mat after her jump. The mat brings her to a stop after 1 second. She was traveling at 5.0 m/sec when she landed on the mat. Note: The speed of the jumper at the top of her jump, before she started to fall toward the mat, was 0 m/sec.
   a. What is the change in momentum for the jumper?
   b. What is the force felt by the jumper upon impact with the mat?

5. A 0.5-kilogram soccer ball is kicked with a force of 50 newtons for 0.2 seconds. The ball was at rest before the kick. What is the speed of the soccer ball after the kick?

6. A baseball player hits a 0.155-kilogram fastball traveling at 44.0 m/sec into center field at a speed of 50.0 m/sec. If the impact lasts for 0.00450 second, with what force does he hit the baseball?

7. Tow Sawyer launches his 180-kilogram raft on the Mississippi River by pushing on it with a force of 75 newtons. How long must Tom push on the raft to accelerate it to a speed of 2.0 m/sec?

8. In terms of impulse, why is the ride much more comfortable when an airplane is flying at constant speed versus when it is taking off or landing?

**Thought questions**

9. In certain martial arts, people practice breaking a piece of wood with the side of their bare hand. Use your understanding of impulse to explain how this can be done without injury to the hand.

10. If identical bullets are shot from a pistol and a rifle, a bullet shot from the rifle will travel at a higher speed than a bullet from the pistol. Why? (Hints: Assume shooting force is the same in each case. The barrel of the rifle is longer than the barrel of the pistol.)
11. Boxers attempt to move with an opponent’s punch when it is thrown. In other words, a boxer moves in the same direction as their opponent's punch. This movement may prevent a knockout blow being delivered by their opponent. Explain how.

12. Show that the relationship between impulse and the change in momentum is another way of stating Newton's second law of motion.

13. Mats in a gym, airbags, and padding in sports uniforms are used to protect people from being injured. Explain why these soft objects used instead of rigid objects using your understanding of impulse and change of momentum.
Momentum Conservation

Just like the third law of motion says that forces are equal and opposite, changes in momentum are equal and opposite. This is because when objects exert forces on each other, their motion is affected.

The law of momentum conservation states that if interacting objects in a system are not acted on by outside forces, the total amount of momentum in the system cannot change.

The formula below can be used to find the new velocities of objects if both keep moving after the collision.

\[
m_1v_1(\text{initial}) + m_2v_2(\text{initial}) = m_1v_3(\text{final}) + m_2v_4(\text{final})
\]

If two objects are initially at rest, the total momentum of the system is zero.

\[
\text{the momentum of a system before a collision } = 0
\]

For the final momentum to be zero, the objects must have equal momenta in opposite directions.

\[
0 = m_1v_3 + m_2v_4 \\
m_1v_3 = -(m_2v_4)
\]

**Examples**

**Example 1:** What is the momentum of a 0.2-kilogram steel ball that is rolling at a velocity of 3.0 m/sec?

\[
\text{momentum} = m \times v = 0.2 \text{ kg} \times \frac{3 \text{ m}}{\text{sec}} = 0.6 \text{ kg} \cdot \frac{m}{\text{sec}}
\]

**Example 2:** You and a friend stand facing each other on ice skates. Your mass is 50 kilograms and your friend’s mass is 60 kilograms. As the two of you push off each other, you move with a velocity of 4 m/sec to the right. What is your friend’s velocity?

<table>
<thead>
<tr>
<th>Looking for</th>
<th>Solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>Your friend’s velocity to the left.</td>
<td>( m_1v_3 = -(m_2v_4) )</td>
</tr>
</tbody>
</table>

**Given**

- Your mass of 50 kg.
- Your friend’s mass of 60 kg.
- Your velocity of 4 m/sec to the right.

**Relationship**

\( m_1v_3 = -(m_2v_4) \)

\[
\frac{200 \text{ kg-m/sec}}{-(60 \text{ kg})} = v_4 \\
-3.33 \text{ m/sec} = v_4
\]

Your friend’s velocity to the left is 3.33 m/sec.
1. If a ball is rolling at a velocity of 1.5 m/sec and has a momentum of 10.0 kg·m/sec, what is the mass of the ball?

2. What is the velocity of an object that has a mass of 2.5 kilogram and a momentum of 1,000 kg·m/sec?

3. Tiger Woods hits 45.0-gram golf ball, giving it a speed of 75.0 m/sec. What momentum has Tiger given to the golf ball?

4. A 400-kilogram cannon fires a 10-kilogram cannonball at 20 m/sec. If the cannon is on wheels, at what velocity does it move backward? (This backward motion is called recoil velocity.)

5. "Big" Al stands on a skateboard at rest and throws a 0.5-kilogram rock at a velocity of 10.0 m/sec. "Big" Al moves back at 0.05 m/sec. What is the combined mass of "Big" Al and the skateboard?

6. As the boat in which he is riding approaches a dock at 3.0 m/sec, Jasper stands up in the boat and jumps toward the dock. Jasper applies an average force of 800 newtons on the boat for 0.30 seconds as he jumps.
   a. How much momentum does Jasper's 80-kilogram body have as it lands on the dock?
   b. What is Jasper's speed on the dock?

7. Daryl the delivery guy gets out of his pizza delivery truck forgetting to set the parking brake. The 2,000 kilogram truck rolls down hill reaching a speed of 30 m/sec just before hitting a large oak tree. The vehicle stops 0.72 seconds after first making contact with the tree.
   a. How much momentum does the truck have just before hitting the tree?
   b. What is the average force applied by the tree?

8. Two billion people jump up in the air at the same time with an average velocity of 7.0 m/sec. If the mass of an average person is 60 kilograms and the mass of Earth is $5.98 \times 10^{24}$ kilograms:
   a. What is the total momentum of the two billion people?
   b. What is the effect of their action on Earth?

9. Tammy, a lifeguard, spots a swimmer struggling in the surf and jumps from her lifeguard chair to the sand beach. She makes contact with the sand at a speed of 6.00 m/sec leaving an indentation in the sand 0.10 meters deep.
   a. If Tammy's mass is 60. kilograms, what is momentum as she first touches the sand?
   b. What is the average force applied on Tammy by the sand beach?

10. When a gun is fired, the shooter describes the sensation of the gun kicking. Explain this in terms of momentum conservation.

11. What does it mean to say that momentum is conserved?
Work

In science, “work” is defined with an equation. Work is the amount of force applied to an object (in the same
direction as the motion) over a distance. By measuring how much force you have used to move something over a
certain distance, you can calculate how much work you have accomplished.

The formula for work is:

\[ W = F \times d \]

An **joule** of work is actually a **newton-meter**; both units represent the same thing: work! In fact, one joule of work
is defined as a force of one newton that is exerted on an object to it a distance of one meter.

\[ 1.0 \text{ joule} = 1.0 \text{ N} \times 1.0 \text{ meter} = 1.0 \text{ newton-meter} \]

### Examples

How much work is done on a 10-newton block that is lifted 5 meters off the ground by a pulley?

<table>
<thead>
<tr>
<th>Looking for</th>
<th>Solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>The amount of work done by a pulley in unit of newtons and meters.</td>
<td>Work = 10 N × 5 m</td>
</tr>
<tr>
<td>The lift force applied by the pulley = 10 N</td>
<td>Work = 50 newton-meters</td>
</tr>
<tr>
<td>The distance the force was lifted = 5 meters.</td>
<td>The pulley did 50 newton-meters or 50 joules of work.</td>
</tr>
</tbody>
</table>

### Practice

1. In your own words, define work in scientific terms. Be complete in your definition.
2. How are work, force, and distance related?
3. What are two different units that represent work?
4. For the following situations, determine whether work was done. Write “work done” or “no work done” for each situation.
   a. An ice skater glides for two meters across ice.
   b. The ice skater’s partner lifts her up a distance of 1 meter.
   c. The ice skater’s partner carries her across the ice a distance of 3 meters.
   d. After setting her down, the ice skater’s partner pulls her across the ice a distance of 10 meters.
   e. After skating practice, the ice skater lifts her 20-newton gym bag up 0.5 meter.
5. A woman lifts her 100-newton child up one meter and carries her for a distance of 50 meters to the child’s bedroom. How much work does the woman do?

6. How much work does a mother do if she lifts each of her twin babies upward 1 meter? Each baby weighs 90 newtons.

7. You pull your sled through the snow a distance of 500 meters with a horizontal force of 200 newtons. How much work did you do?

8. Because the snow suddenly gets too slushy, you decide to carry your 100-newton sled the rest of the way home. How much work do you do when you pick up the sled, lifting it 0.5 meter upward? How much work do you do to carry the sled if your house is 800 meters away?

9. An ant sits on the back of a mouse. The mouse carries the ant across the floor for a distance of 10 meters. Was there work done by the mouse? Explain.

10. You decide to add up all the work you did yesterday. If you accomplished 10,000 newton-meters of work yesterday, how much work did you do in units of joules?

11. You did 150 joules of work lifting a 120-newton backpack.
   a. How high did you lift the backpack?
   b. How much did the backpack weigh in pounds? (Hint: There are 4.448 newtons in one pound.)

12. A crane does 62,500 joules of work to lift a boulder a distance of 25.0 meters. How much did the boulder weigh? (Hint: The weight of an object is considered to be a force in units of newtons.)

13. A bulldozer does 30,000 joules of work to push another boulder a distance of 20 meters. How much force is applied to push the boulder?

14. You lift a 45-newton bag of mulch 1.2 meters and carry it a distance of 10 meters to the garden. How much work was done?

15. A 450-newton gymnast jumps upward a distance of 0.50 meters to reach the uneven parallel bars. How much work did she do before she even began her routine?

16. It took a 500.0-newton ballerina a force of 250 joules to lift herself upward through the air. How high did she jump?

17. A people-moving conveyor-belt moves a 600-newton person a distance of 100 meters through the airport.
   a. How much work was done?
   b. The same 600-newton person lifts his 100-newton carry-on bag upward a distance of 1 meter. They travel another 10 meters by riding on the “people mover.” How much work was done in this situation?

18. Which person did the most work?
   a. John walks 1,000 meters to the store. He buys 4.448 newtons of candy and then carries it to his friend’s house which is 500 meters away.
   b. Sally lifts her 22-newton cat a distance of 0.5 meter.
   c. Henry carries groceries from a car to his house. Each bag of groceries weighs 40 newtons. He has ten bags. He lifts each bag up one meter to carry it and then walks 10 meters from his car to his house.
Potential and Kinetic Energy

Potential energy is stored energy. The formula for the potential energy of an object is:  

$$E_p = mgh$$

where $m$ equals mass in kilograms, $g$ is the acceleration of gravity, and $h$ equals the height of the object. The mass ($m$) of the object times the acceleration of gravity ($g$) is the same as the weight of the object in newtons. The acceleration of gravity is equal to 9.8 m/sec$^2$.

$$\text{mass of the object (kilograms)} \times \frac{9.8 \text{ m}}{\text{sec}^2} = \text{weight of the object (newtons)}$$

Kinetic energy is energy of motion. The formula for the kinetic energy of an object is:  

$$E_k = \frac{1}{2}mv^2$$

where $m$ equals mass in kilograms and $v$ equals the velocity or speed of the object in meters per second. To do this calculation, square the velocity value. Next, multiply by the mass, and then, divide by 2.

Energy is measured in joules or newton-meters.

$$1 \text{ N} = 1 \text{ kg} \cdot \frac{m}{\text{sec}^2}$$

$$1 \text{ joule} = 1 \text{ kg} \cdot \frac{m^2}{\text{sec}^2} = 1 \text{ N} \cdot \text{m}$$

**EXAMPLES**

**Example 1:** A 50-kilogram boy and his 100-kilogram father went jogging. Both ran at a rate of 5 m/sec. Who had more kinetic energy?

Although the boy and his father were running at the same speed, the father has more kinetic energy because he has more mass.  

The kinetic energy of the boy:  

$$E_k = \frac{1}{2}mv^2 = \frac{1}{2}(50 \text{ kg})(5 \text{ m/sec})^2 = 625 \text{ kg} \cdot \frac{m^2}{\text{sec}^2}$$

The kinetic energy of the father:  

$$E_k = \frac{1}{2}mv^2 = \frac{1}{2}(100 \text{ kg})(5 \text{ m/sec})^2 = 1,250 \text{ kg} \cdot \frac{m^2}{\text{sec}^2}$$

**Example 2:** What is the potential energy of a 10-newton book that is placed on a shelf that is 2.5 meters high?

The book’s weight (10 newtons) is equal to its mass times the acceleration of gravity. Therefore, you can easily use this value in the potential energy formula:

$$E_p = mgh = (10 \text{ N})(2.5 \text{ m}) = 25 \text{ N} \cdot \text{m}$$

**PRACTICE**

Show all calculations. Write all energy values in units of joules. One joule = one newton-meter

1. What is the potential energy of a 2-kilogram potted plant that is on a 1 meter-high plant stand?
2. What is the kinetic energy of a 3-kilogram ball that is rolling at 2 meters per second?

3. The potential energy of an apple is 6.00 joules. The apple is 3.00-meters high. What is the mass of the apple?

4. Determine the amount of potential energy of a 5-newton book that is moved to three different shelves on a bookcase. The height of each shelf is 1.0 meter, 1.5 meters, and 2.0 meters.

5. Two objects were lifted by a machine. One object had a mass of 2 kilograms, and was lifted at a speed of 2 m/sec. The other had a mass of 4 kilograms and was lifted at a rate of 3 m/sec.
   a. Which object had more kinetic energy while it was being lifted?
   b. Which object had more potential energy when it was lifted to a distance of 10 meters? Show your calculation. (Remember that gravity = 9.8 m/sec²)

6. You are on roller blades on top of a small hill. Your potential energy is equal to 1,000.0 joules. The last time you checked your mass was 60.0 kilograms.
   a. What is your weight in newtons?
   b. What is the height of the hill?
   c. If you start skating down this hill, your potential energy will be converted to kinetic energy. At the bottom of the hill, your kinetic energy will be equal to your potential energy at the top. What will be your speed at the bottom of the hill?

7. Answer the following:
   a. What is the kinetic energy of a 1-kilogram ball is thrown into the air with an initial velocity of 30 m/sec?
   b. How much potential energy does the ball have when it reaches the top of its ascent?
   c. How high into the air did the ball travel?

8. What is the potential energy of a 3 kilogram-ball that is on the ground?

9. What is the kinetic energy of a 2,000-kilogram boat moving at 5 m/sec?

10. What is the velocity of an 500-kilogram elevator that has 4,000 joules of energy?

11. What is the mass of an object that creates 33,750 joules of energy by traveling at 30 m/sec?

12. In a lab investigation, one group of students (group A) measures the speed of a 0.1-kilogram car at 2.5 m/sec at the bottom of a hill. Another group of students (group B) measures the speed of the car at 3 m/sec at the bottom of the hill. The car’s starting position at the top of the hill is one-meter high.
   a. What is the potential energy of the car at the beginning of the experiment before its speed is measured?
   b. Calculate the kinetic energy of the car for group A using the speed (2.5 m/sec) and mass values above.
   c. Calculate the kinetic energy of the car for group B using the speed (3.0 m/sec) and mass values above.
   d. At the bottom of a hill, the kinetic energy of the cars should be equal to the potential energy of the car at the top of the hill. Are the kinetic energy values for groups A and B equal to, less than, or greater than the potential energy value?
   e. The energy of an object can be converted to heat due to the friction of the car on the hill. The difference between the potential energy of the car and its kinetic energy at the bottom of the hill equals the energy lost due to friction. How much energy is lost due to heat for group A’s car? How much for group B’s car?
Collisions and Conservation of Momentum

There are two main types of collisions: elastic and inelastic.

As long as there are no outside forces (such as friction), momentum is conserved in both elastic and inelastic collisions.

Conservation of momentum makes it possible to determine the motion of objects before and after colliding.

The steps from the text for using momentum to solve collision problems are provided in the graphic below. Use these problem-solving steps and the problem-solving table to complete this skill sheet. Keep in mind that bounces have greater momentum change.

Problem Solving Steps

1. Draw a diagram
2. Decide whether the collision is elastic or inelastic
3. Assign variables to represent the masses and velocities of the objects before and after the collision.
4. Use momentum conservation to write an equation stating that the total momentum before the collision equals the total after.
   Then solve it.

Example

A 2,000-kilogram railroad car moving at 5 m/sec collides with a 6,000-kilogram railroad car at rest. If the cars coupled together, what is their velocity after the is inelastic collision?

Looking for

\( m_3 = \text{the velocity of the combined railroad cars after an inelastic collision} \)

Given

Initial speed and mass of both cars:

\( m_1 = 2,000 \text{ kg}, v_1 = 5 \text{ m/sec} \)

\( m_2 = 6,000 \text{ kg}, v_2 = 0 \text{ m/sec} \)

Combined mass of the two cars:

\( m_1 + m_2 = 8,000 \text{ kg} \)

Solution

\( (2000 \text{ kg})(5 \text{ m/sec}) + (6000 \text{ kg})(0 \text{ m/sec}) = (2000 \text{ kg} + 6000 \text{ kg})v_3 \)

\( 10,000 \text{ kg-m/sec} = (8000 \text{ kg})v_3 \)

\( \frac{10,000 \text{ kg-m/sec}}{8000 \text{ kg}} = v_3 \)

\( 10 \text{ m/sec} = v_3 \)

The velocity of the two combined cars after the collision is 10 m/sec.
1. What is the momentum of a 100-kilogram fullback carrying a football on a play at a velocity of 3.5 m/sec.

2. What is the momentum of a 75.0-kilogram defensive back chasing the fullback at a velocity of 5.00 m/sec.

3. A 2,000-kilogram railroad car moving at 5 m/sec to the east collides with a 6,000-kilogram railroad car moving at 3 m/sec to the west. If the cars couple together, what is their velocity after the collision?

4. A 4-kilogram ball moving at 8 m/sec to the right collides with a 1-kilogram ball at rest. After the collision, the 4-kilogram ball moves at 4.8 m/sec to the right. What is the velocity of the 1-kilogram ball?

5. A 0.0010-kg pellet is fired at a speed of 50.0 m/s at a motionless 0.35-kg piece of balsa wood. When the pellet hits the wood, it sticks in the wood and they slide off together. With what speed do they slide?

6. Terry, a 70-kilogram tailback, runs through his offensive line at a speed of 7.0 m/sec. Jared, a 100-kilogram linebacker, running in the opposite direction at 6.0 m/s, meets Jared head-on and “wraps him up.” What is the result of this tackle?

7. Snowboarding cautiously down a steep slope at a speed of 7.0 m/sec, Sarah, whose mass is 50. kilograms, is afraid she won't have enough speed to travel up a slight uphill grade ahead of her. She extends her hand as her friend Trevor, having a mass of 100. kilograms is about to pass her traveling at 16 m/sec. If Trevor grabs her hand, calculate the speed at which the friends will be sliding.

8. Tex, an 85.0 kilogram rodeo bull rider is thrown from the bull after a short ride. The 520. kilogram bull chases after Tex at 13.0 m/sec. While running away at 3.00 m/sec, Tex jumps onto the back of the bull to avoid being trampled. How fast does the bull run with Tex aboard?

9. Identical twins Kate and Karen are rowing their boat on a hot Summer afternoon when they decide to go for a swim. Kate, whose mass is 45 kilograms, jumps off the front of the boat at a speed of 3.00 m/sec. Karen jumps off the back at a speed of 4.00 m/sec. If the 70-kilogram rowboat is moving at 1.00 m/s when the girls jump, what is the speed of the rowboat after the girls jump?

10. A 0.10-kilogram piece of modeling clay is tossed at a motionless 0.10-kilogram block of wood and sticks. The block slides across a frictionless table at 15 m/sec.
   a. At what speed was the clay tossed?
   b. The clay is replaced with a “bouncy” ball tossed with the same speed. The bouncy ball rebounds from the wooden block at a speed of 10 meters per second. What effect does this have on the wooden block? Why?
Any time you lift an object, you do work against gravity. We use the same formula for work that you already know (Work = force × distance), but it’s expressed in a slightly different form:

\[
\text{Work against gravity} = \text{mass} \times \text{acceleration due to gravity} \times \text{height} \\
W = mgh
\]

Force is written in the form \( mg \), where \( m \) is mass and \( g \) is the acceleration due to gravity, 9.8 m/sec\(^2\). We use \( h \) for height because only the vertical distance an object moves matters for calculating work against gravity.

Did you know...If you have to lift a new sofa to a second-floor apartment, the work done against gravity is the same whether you haul it straight up the side of the building with ropes or take a longer path up the stairs. Only the vertical distance matters because the force of gravity is vertical.

**EXAMPLE**

You lift a 2-liter bottle of cola from a grocery bag on the floor to a refrigerator shelf that is 0.8 meter high. If the bottle has a mass of 2.02 kilograms, how much work did you do against gravity?

### Looking for
The amount of work done against gravity.

### Given
- mass of bottle = 2.02 kilograms
- acceleration due to gravity = 9.8 m/sec\(^2\)
- height = 0.8 meter

### Relationship
\( W = mgh \)

### Solution
\[
W = mgh \\
W = 2.02 \text{ kg} \times 9.8 \text{ m/sec}^2 \times 0.8 \text{ m} \\
W = 15.8 \text{ joules}
\]

**PRACTICE**

1. Jai-Anna, who has a mass of 45 kilograms, climbed 3 meters up a ladder to rescue her cat from a tree. How much work against gravity did she do?
2. A tram inside the Gateway Arch in Saint Louis, Missouri lifts visitors to a window-lined observation room 192 meters above the ground. How much work does the tram’s motor do against gravity to carry two 55-kilogram passengers to this room? (You may ignore the work done by the motor to carry the tram itself).
3. You pick up a 10-newton book off the floor and put it on a shelf 2 meters high. How much work did you do?
4. Elijah does 44 joules of work against gravity to pull a 0.5-kilogram rope with a 1.0-kilogram bucket attached up to the floor of his tree house. How many meters high is his tree house?
5. Alejandra weighs 225 newtons. How much work does she do against gravity when she climbs to a ledge at the top of a 15-meter climbing wall?
6. A window-washer stands on a scaffolding 30 meters above the ground. If he did 23,520 joules of work to reach the scaffolding, what is his mass?
Power

In science, work is defined as the force needed to move an object a certain distance. The amount of work done per unit of time is called power.

**EXAMPLE**

Suppose you and a friend are helping a neighbor to reshingle the roof of his home. You each carry 10.0 bundles of shingles weighing 300 newtons apiece up to the roof which is 7.00 meters from the ground. You are able to carry the shingles to the roof in 10.0 minutes but your friend needs 20.0 minutes.

Both of you did the same amount of work (force \( \times \) distance) but you did the work in a shorter time.

\[
W = F \times d
\]

\[
W = 10 \text{ bundles of shingles} \times (300 \text{ N/bundle}) \times 7.00 \text{ m} = 21,000 \text{ joules}
\]

However, you had more power than your friend.

\[
\text{Power (watts)} = \frac{\text{Work (joules)}}{\text{Time (seconds)}}
\]

Let’s do the math to see how this is possible.

**Step one:** Convert minutes to seconds.

\[
10 \text{ minutes} \times \frac{60 \text{ seconds}}{\text{minute}} = 600 \text{ seconds (You)}
\]

\[
20 \text{ minutes} \times \frac{60 \text{ seconds}}{\text{minute}} = 1,200 \text{ seconds (Friend)}
\]

**Step two:** Find power.

\[
\frac{21,000 \text{ joules}}{600 \text{ seconds}} = 35 \text{ watts (You)}
\]

\[
\frac{21,000 \text{ joules}}{1,200 \text{ seconds}} = 17.5 \text{ watts (Friend)}
\]

As you can see, the same amount of work that is done in less time produces more power. You are familiar with the word *watt* from a light bulb. Is it now clear to you why a 100-watt bulb is more powerful than a 40-watt bulb?
1. A motor does 5,000 joules of work in 20 seconds. What is the power of the motor?

2. A machine does 1,500 joules of work in 30 seconds. What is the power of this machine?

3. A hair dryer uses 72,000 joules of energy in 60 seconds. What is the power of this hair dryer?

4. A toaster oven uses 67,500 joules of energy in 45 seconds to toast a piece of bread. What is the power of the oven?

5. A horse moves a sleigh 1.00 kilometer by applying a horizontal 2,000-newton force on its harness for 45 minutes. What is the power of the horse? (Hint: Convert time to seconds.)

6. A wagon is pulled at a speed of 0.40 meters/sec by a horse exerting an 1,800-newton horizontal force. What is the power of this horse?

7. Suppose a force of 100 newtons is used to push an object a distance of 5 meters in 15 seconds. Find the work done and the power for this situation.

8. Emily’s vacuum cleaner has a power rating of 200 watts. If the vacuum cleaner does 360,000 joules of work, how long did Emily spend vacuuming?

9. Nicholas spends 20 minutes ironing shirts with his 1,800-watt iron. How many joules of energy were used by the iron? (Hint: convert time to seconds).

10. It take a clothes dryer 45 minutes to dry a load of towels. If the dryer uses 6,750,000 joules of energy to dry the towels, what is the power rating of the machine?

11. A 1000-watt microwave oven takes 90 seconds to heat a bowl of soup. How many joules of energy does it use?

12. A force of 100 newtons is used to move an object a distance of 15 meters with a power of 25 watts. Find the work done and the time it takes to do the work.

13. If a small machine does 2,500 joules of work on an object to move it a distance of 100 meters in 10 seconds, what is the force needed to do the work? What is the power of the machine doing the work?

14. A machine uses a force of 200 newtons to do 20,000 joules of work in 20 seconds. Find the distance the object moved and the power of the machine. (Hint: A joule is the same as a Newton-meter.)

15. A machine that uses 200 watts of power moves an object a distance of 15 meters in 25 seconds. Find the force needed and the work done by this machine.
# Mechanical Advantage

Mechanical advantage (MA) is the ratio of output force to input force for a machine.

\[ MA = \frac{F_o}{F_i} \]

or

\[ MA = \frac{\text{output force (N)}}{\text{input force (N)}} \]

Did you notice that the force unit involved in the calculation, the newton (N) is present in both the numerator and the denominator of the fraction? These units cancel each other, leaving the value for mechanical advantage unitless.

\[ \frac{\text{newtons}}{\text{newtons}} = \frac{N}{N} = 1 \]

Mechanical advantage tells you how many times a machine multiplies the force put into it. Some machines provide us with more output force than we applied to the machine—this means MA is greater than one. Some machines produce an output force smaller than our effort force, and MA is less than one. We choose the type of machine that will give us the appropriate MA for the work that needs to be performed.

### Examples

**Example 1:** A force of 200 newtons is applied to a machine in order to lift a 1,000-newton load. What is the mechanical advantage of the machine?

\[ MA = \frac{\text{output force}}{\text{input force}} = \frac{1000 \text{ N}}{200 \text{ N}} = 5 \]

Machines make work easier. Work is force times distance \((W = F \times d)\). The unit for work is the newton-meter. Using the work equation, as shown in example 2 below, can help calculate the mechanical advantage.

**Example 2:** A force of 30 newtons is applied to a machine through a distance of 10 meters. The machine is designed to lift an object to a height of 2 meters. If the total work output for the machine is 18 newton-meters (N-m), what is the mechanical advantage of the machine?

\[ \text{input force} = 30 \text{ N} \quad \text{output force} = (\text{work} \div \text{distance}) = (18 \text{ N-m} \div 2 \text{ m}) = 9 \text{ N} \]

\[ MA = \frac{\text{output force}}{\text{input force}} = \frac{9 \text{ N}}{30 \text{ N}} = 0.3 \]
1. A machine uses an input force of 200 newtons to produce an output force of 800 newtons. What is the mechanical advantage of this machine?

2. Another machine uses an input force of 200 newtons to produce an output force of 80 newtons. What is the mechanical advantage of this machine?

3. A machine is required to produce an output force of 600 newtons. If the machine has a mechanical advantage of 6, what input force must be applied to the machine?

4. A machine with a mechanical advantage of 10 is used to produce an output force of 250 newtons. What input force is applied to this machine?

5. A machine with a mechanical advantage of 2.5 requires an input force of 120 newtons. What output force is produced by this machine?

6. An input force of 35 newtons is applied to a machine with a mechanical advantage of 0.75. What is the size of the load this machine could lift (how large is the output force)?

7. A machine is designed to lift an object with a weight of 12 newtons. If the input force for the machine is set at 4 newtons, what is the mechanical advantage of the machine?

8. An input force of 50 newtons is applied through a distance of 10 meters to a machine with a mechanical advantage of 3. If the work output for the machine is 450 newton-meters and this work is applied through a distance of 3 meters, what is the output force of the machine?

9. Two hundred newton-meters of work is put into a machine over a distance of 20 meters. The machine does 150 newton-meters of work as it lifts a load 10 meters high. What is the mechanical advantage of the machine?

10. A machine has a mechanical advantage of 5. If 300 newtons of input force is used to produce 3,000 newton-meters of work,
    a. What is the output force?
    b. What is the distance over which the work is applied?
Mechanical Advantage of Simple Machines

We use simple machines to make tasks easier. While the output work of a simple machine can never be greater than the input work, a simple machine can multiply input forces OR multiply input distances (but never both at the same time). You can use this skill sheet to practice calculating mechanical advantage (MA) for two common simple machines: levers and ramps.

The general formula for the mechanical advantage (MA) of levers:

\[ MA_{\text{lever}} = \frac{F_o}{F_i} \] (output force) (input force)

Or you can use the ratio of the input arm length to the output arm length:

\[ MA_{\text{lever}} = \frac{L_i}{L_o} \] (length of input arm) (length of output arm)

Most of the time, levers are used to multiply force to lift heavy objects.

The general formula for the mechanical advantage (MA) of ramps:

\[ MA_{\text{ramp}} = \frac{\text{ramp length}}{\text{ramp height}} \]

A ramp makes it possible to move a heavy load to a new height using less force (but over a longer distance). The mechanical advantage of a ramp can be found using this formula:

**EXAMPLES**

**Example 1:** A construction worker uses a board and log as a lever to lift a heavy rock. If the input arm is 3 meters long and the output arm is 0.75 meters long, what is the mechanical advantage of the lever?

\[ MA = \frac{3 \text{ meters}}{0.75 \text{ meter}} = 4 \]

**Example 2:** Sometimes levers are used to multiply distance. For a broom, your upper hand is the fulcrum and your lower hand provides the input force: Notice the input arm is shorter than the output arm. The mechanical advantage of this broom is:

\[ MA = \frac{0.3 \text{ meter}}{1.2 \text{ meters}} = 0.25 \]

A mechanical advantage less than one doesn’t mean a machine isn’t useful. It just means that instead of multiplying force, the machine multiplies distance. A broom doesn’t push the dust with as much force as you use to push the broom, but a small movement of your arm pushes the dust a large distance.
Example 3: A 500-newton cart is lifted to a height of 1 meter using a 10-meter long ramp. You can see that the worker only has to use 50 newtons of force to pull the cart. You can figure the mechanical advantage in either of these two ways:

\[ MA_{\text{ramp}} = \frac{\text{ramp length}}{\text{ramp height}} = \frac{10 \text{ meters}}{1 \text{ meter}} = 10 \]

Or using the standard formula for mechanical advantage:

\[ MA = \frac{\text{output force}}{\text{input force}} = \frac{500 \text{ newtons}}{50 \text{ newtons}} = 10 \]

Lever problems

1. A lever used to lift a heavy box has an input arm of 4 meters and an output arm of 0.8 meters. What is the mechanical advantage of the lever?

2. What is the mechanical advantage of a lever that has an input arm of 3 meters and an output arm of 2 meters?

3. A lever with an input arm of 2 meters has a mechanical advantage of 4. What is the output arm’s length?

4. A lever with an output arm of 0.8 meter has a mechanical advantage of 6. What is the length of the input arm?

5. A rake is held so that its input arm is 0.4 meters and its output arm is 1.0 meters. What is the mechanical advantage of the rake?

6. A broom with an input arm length of 0.4 meters has a mechanical advantage of 0.5. What is the length of the output arm?

7. A child’s toy rake is held so that its output arm is 0.75 meters. If the mechanical advantage is 0.33, what is the input arm length?
Ramp problems

8. A 5-meter ramp lifts objects to a height of 0.75 meters. What is the mechanical advantage of the ramp?

9. A 10-meter long ramp has a mechanical advantage of 5. What is the height of the ramp?

10. A ramp with a mechanical advantage of 8 lifts objects to a height of 1.5 meters. How long is the ramp?

11. A child makes a ramp to push his toy dump truck up to his sandbox. If he uses 5 newtons of force to push the 12-newton truck up the ramp, what is the mechanical advantage of his ramp?

12. A ramp with a mechanical advantage of 6 is used to move a 36-newton load. What input force is needed to push the load up the ramp?

13. Gina wheels her wheelchair up a ramp using a force of 80 newtons. If the ramp has a mechanical advantage of 7, what is the output force (in newtons)?

14. **Challenge!** A mover uses a ramp to pull a 1000-newton cart up to the floor of his truck (0.8 meters high). If it takes a force of 200 newtons to pull the cart, what is the length of the ramp?
**Gear Ratios**

A gear ratio is used to figure out the number of turns each gear in a pair will make based on the number of teeth each gear has.

To calculate the gear ratio for a pair of gears that are working together, you need to know the number of teeth on each gear. The formula below demonstrates how to calculate a gear ratio.

\[ \frac{T_o}{T_i} = \frac{N_i}{N_o} \]

Notice that knowing the number of teeth on each gear allows you to figure out how many turns each gear will take.

Why would this be important in figuring out how to design a clock that has a minute and hour hand?

**EXAMPLE**

A gear with 48 teeth is connected to a gear with 12 teeth. If the 48-tooth gear makes one complete turn, how many times will the 12-tooth gear turn?

\[
\frac{T_o}{T_i} = \frac{48}{12} = 4 \text{ turns}
\]

**PRACTICE**

1. A 36-tooth gear turns three times. It is connected to a 12-tooth gear. How many times does the 12-tooth gear turn?
2. A 12-tooth gear is turned two times. How many times will the 24-tooth gear to which it is connected turn?
3. A 60-tooth gear is connected to a 24-tooth gear. If the smaller gear turns ten times, how many turns does the larger gear make?
4. A 60-tooth gear is connected to a 72-tooth gear. If the smaller gear turns twelve times, how many turns does the larger gear make?
5. A 72-tooth gear is connected to a 12-tooth gear. If the large gear makes one complete turn, how many turns does the small gear make?
6. Use the gear ratio formula to help you fill in the table below.

**Table 1: Using the gear ratio to calculate number of turns**

<table>
<thead>
<tr>
<th>Input Gear (# of teeth)</th>
<th>Output Gear (# of teeth)</th>
<th>Gear ratio (Input Gear: Output Gear)</th>
<th>How many turns does the output gear make if the input gear turns 3 times?</th>
<th>How many turns does the input gear make if the output gear turns 2 times?</th>
</tr>
</thead>
<tbody>
<tr>
<td>24</td>
<td>24</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>36</td>
<td>12</td>
<td>3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>24</td>
<td>36</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>48</td>
<td>36</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>24</td>
<td>48</td>
<td>1</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

7. The problems in this section involve three gears stacked on top of each other. Once you have filled in Table 2, answer the question that follow. Use the gear ratio formula to help. Remember, knowing the gear ratios allows you to figure out the number of turns for a pair of gears.

**Table 2: Set up for three gears**

<table>
<thead>
<tr>
<th>Set up</th>
<th>Gears</th>
<th>Number of teeth</th>
<th>Ratio (top gear: middle gear)</th>
<th>Ratio 2 (middle gear: bottom gear)</th>
<th>Total gear ratio (Ratio 1 x Ratio 2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Top gear: 12</td>
<td>12</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Middle gear: 24</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Bottom gear: 36</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Top gear: 24</td>
<td>24</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Middle gear: 36</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Bottom gear: 12</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Top gear: 12</td>
<td>12</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Middle gear: 48</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Bottom gear: 24</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>Top gear: 24</td>
<td>24</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Middle gear: 48</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Bottom gear: 36</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

8. As you turn the top gear to the right, what direction does the middle gear turn? What direction will the bottom gear turn?

9. How many times will you need to turn the top gear (input) in set up 1 to get the bottom gear (output) to turn once?

10. If you turn the top gear (input) in set up 2 two times, how many times will the bottom gear (output) turn?

11. How many times will the middle gear (output) in set up 3 turn if you turn the top gear (input) two times?

12. How many times will you need to turn the top gear (input) in set up 4 to get the bottom gear (output) to turn 4 times?
Efficiency

In a perfect machine, the work input would equal the work output. However, there aren’t any perfect machines in our everyday world. Bicycles, washing machines, and even pencil sharpeners lose some input work to friction. Efficiency is the ratio of work output to work input. It is expressed as a percent. A perfect machine would have an efficiency of 100 percent.

**Efficiency of the first design**

\[
\text{Efficiency} = \frac{\text{work output}}{\text{work input}}
\]

\[
\frac{10 \text{ joules}}{20 \text{ joules}} = 50\%
\]

**Efficiency of the second design**

\[
\text{Efficiency} = \frac{\text{work output}}{\text{work input}}
\]

\[
\frac{13 \text{ joules}}{20 \text{ joules}} = 65\%
\]

The second design is 15\% more efficient than the first.

**Practice**

1. A cell phone charger uses 4.83 joules per second when plugged into an outlet, but only 1.31 joules per second actually goes into the cell phone battery. The remaining joules are lost as heat. That’s why the battery feels warm after it has been charging for a while. How efficient is the charger?

2. A professional cyclist rides a bicycle that is 92 percent efficient. For every 100 joules of energy he exerts as input work on the pedals, how many joules of output work are used to move the bicycle?

3. An automobile engine is 15\% efficient. How many joules of input work are required to produce 15,000 joules of output work to move the car?

4. It takes 56.5 kilojoules of energy to raise the temperature of 150 milliliters of water from 5°C to 95°C. If you use an electric water heater that is 60\% efficient, how many kilojoules of electrical energy will the heater actually use by the time the water reaches its final temperature?

5. A power station burns 75 kilograms of coal per second. Each kg of coal contains 27 million joules of energy.
   a. What is the total power of this power station in watts? (watts = joules/second)
   b. The power station’s output is 800 million watts. How efficient is this power station?

6. A machine requires 2,000 joules to raise a 20 kilogram block a distance of 6 meters. How efficient is the machine? (Hint: Work done against gravity = mass × acceleration due to gravity × height.)
**Equilibrium**

When all forces acting on a body are balanced, the forces are in equilibrium. Here are free-body diagrams for you to use for practice working with equilibrium.

Remember that an unbalanced force results in acceleration. Therefore, the forces acting on an object that is not accelerating must be at balanced. These objects may be at rest, or they could be moving at a constant velocity. Either way, we say that the forces acting on these objects are in *equilibrium*.

**Example**

What force is necessary in the free-body diagram at right to achieve equilibrium?

<table>
<thead>
<tr>
<th>Looking for</th>
<th>Solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>The unknown force: (? N)</td>
<td>[600 \text{ N} = 400 \text{ N} + ? \text{ N}]</td>
</tr>
</tbody>
</table>

**Given**

- 600 N is pressing down on the box.
- 400 N is pressing up on the box.

**Relationship**

You can solve equilibrium problems using simple equations:

\[600 \text{ N} = 400 \text{ N} + ? \text{ N}\]

**Practice**

For each free-body diagram, supply the force or forces necessary to achieve equilibrium.

1. Draw a force arrow, and write in the force.

2. Supply the missing force.
3. Distribute the unknown forces evenly to prevent rotation.

4. Supply the missing forces.

Think about these

5. Here is the classic “asteroid destroys Earth” scenario. The momentum of the asteroid is beyond the forces that even thermonuclear bombs might apply to stop its approach. Assuming that you can apply only modest forces, where might they best be applied to result in a new acceleration that will, as they say, “save the world”? Draw an arrow to show the best location on the asteroid to apply force so that it avoids hitting Earth.

6. Helium balloons stay the same size as you hold them, but swell and burst as they rise to high altitudes when you let them go. Draw and label force arrows inside and/or outside the balloons on the graphic at right to show why the near Earth balloon does not burst, but the high altitude balloon does eventually burst. Hint: What are the forces on the inside of the balloon? What are the forces on the outside of the balloons?
In this skill sheet, you will practice solving problems that involve torque. Torque is an action that is created by an applied force and causes an object to rotate. Any object that rotates has a torque associated with it.

Torque, \( \tau \), can be calculated by multiplying the applied force, \( F \), by \( r \). The value, \( r \), is the perpendicular distance between the point of rotation and the line of action of the force (the line along which the force is applied).

\[
\tau = F \times r
\]

The unit of torque is newton·meter (N·m).

For many situations the distance \( r \) is also called the lever arm.

A see-saw works based on torque. As you know, the lighter person (or a cat!) has to sit further out for the see-saw to be level. You know now that this is because the only way to make the torque of the heavy person equal to the torque of the light person is to increase the lever arm of the light person.

**Example**

For an object to be in rotational equilibrium about a certain point, the total torque about this point must be zero.

For the example shown in the figure, calculate the magnitude and direction of a force that must be applied at point B for rotational equilibrium about point P.

<table>
<thead>
<tr>
<th>Looking for</th>
<th>Solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>Magnitude and force that must be applied at point B for rotational equilibrium.</td>
<td>( F_{\text{left}} \times r_{\text{left}} = F_{\text{right}} \times r_{\text{right}} )</td>
</tr>
<tr>
<td>Given</td>
<td>( F_{\text{left}} \times 0.25 \text{ m} = 40 \text{ N} \times 0.75 \text{ m} )</td>
</tr>
<tr>
<td>Let the force at point B equal ( F_{\text{left}} ).</td>
<td>( F_{\text{left}} = \frac{30 \text{ N} \cdot \text{m}}{0.25 \text{ m}} )</td>
</tr>
<tr>
<td>( r_{\text{left}} = 0.25 \text{ meter} )</td>
<td>( F_{\text{left}} = 120 \text{ N} )</td>
</tr>
<tr>
<td>( F_{\text{right}} = 40 \text{ newtons} )</td>
<td>For rotational equilibrium, 120 N must be applied downward at point B.</td>
</tr>
<tr>
<td>( r_{\text{right}} = 0.75 \text{ meter} )</td>
<td></td>
</tr>
</tbody>
</table>
1. A 10-kilogram mass is suspended from the end of a beam that is 1.2 meters long. The beam is attached to a wall. Find the magnitude and direction (clockwise or counterclockwise) of the resulting torque at point B. Hint: Remember that force is measured in newtons, not kilograms.

2. Two masses $m_1$ and $m_2$ are suspended on an ornament. The ornament is hung from the ceiling at a point which is 10 centimeters from mass $m_1$ and 30 centimeters from mass $m_2$.
   a. If $m_1 = 6$ kg, what does $m_2$ have to be for the ornament to be in rotational equilibrium?
   b. Calculate the ratio of $\frac{m_1}{m_2}$ so that the ornament will be horizontal.
   c. Suppose $m_1 = 10$ kg and $m_2 = 2$ kg. You wish to place a third mass, $m_3 = 5$ kg, on the ornament to make it balance. Should $m_3$ be placed to the right or to the left of the ornament’s suspension point? Explain your answer.
   d. Calculate the exact location where $m_3$ should be placed.

3. Forces are applied on the beam as shown on the figure at right.
   a. Find the torque about point P produced by each of the three forces.
   b. Find the net torque about point P.
   c. A fourth force is applied to the beam at a distance of 0.30 m to the right of point P. What must the magnitude and direction of this force be to make the beam in rotational equilibrium?
Pythagorean Theorem

When you know the x- and y-components of a vector, you can find its magnitude using the Pythagorean theorem. This useful theorem states that $a^2 + b^2 = c^2$, where $a$, $b$, and $c$ are the lengths of the sides of any right triangle.

For example, suppose you need to know the distance represented by the displacement vector $(4, 3)$ m. If you walked east 4 meters then north 3 meters, you would walk a total of 7 meters. This is a distance, but it is not the distance specified by the vector, which describes the shortest way to go. The vector $(4, 3)$ m describes a single straight line. The length of the line is 5 meters because $4^2 + 3^2 = 5^2$.

The Pythagorean theorem can be used to help us calculate the magnitude of a vector once we know its components along the x- and y-directions. Also, we can find one of the components of the vector if we know the other component and the magnitude of the vector.

**EXAMPLE**

A displacement vector $\vec{x} = (2, 3)$ m has these components:
- 2 meters in the x direction.
- 3 meters in the y direction.

What is the magnitude of the vector?

Using the Pythagorean theorem, $a$ is the component along the x direction and $b$ is the component along the y direction. The magnitude of the vector is $c$. We can find the magnitude by taking the square root of $a^2 + b^2$:

$$c = \sqrt{a^2 + b^2}$$

For $a = 2$ m and $b = 3$ m:

$$c = \sqrt{(2 \text{ m})^2 + (3 \text{ m})^2} = \sqrt{13 \text{ m}^2} = 3.6 \text{ m}$$

**PRACTICE**

1. Find the magnitude of the vector $\vec{a} = (3, 4)$.
2. Find the magnitude of the vector $\vec{b} = (-3, -4)$.
3. Find the magnitude of the vector $\vec{z} = (5, 0)$.
4. Find the magnitude of the vector $\vec{x} = (12.00, 6.00)$ cm.
5. A robot starts from a certain point and moves east for a distance of 5.0 meters, then goes north for 3.0 meters, and then turns west for 2.0 meters.
   a. What are the x-y coordinates for the resultant vector?
   b. What is the magnitude of the resultant vector for the robot?

Challenge problems

6. Express the resultant vector in problem 5 above in polar coordinates (magnitude, angle). Assume that the positive x direction is from west to east and the positive y direction is from south to north.

7. A resultant vector has a magnitude of 25 meters. Its y component is –12 meters. What are its two possible x components?

8. For these vectors: $\vec{v}_1 = (5,0), \vec{v}_2 = (0,-3),$ and $\vec{v}_3 = (1,0)$
   a. Add the vectors.
   b. Find the magnitude of the resultant vector.

9. For these vectors: $\vec{v}_1 = (-5,0), \vec{v}_2 = (0,-2),$ and $\vec{v}_3 = (7,0)$
   a. Add the vectors.
   b. Find the magnitude of the resultant vector.

10. For these vectors: $\vec{v}_1 = (5,0), \vec{v}_2 = (0,-5),$ and $\vec{v}_3 = (5,180^\circ)$
    a. Add the vectors.
    b. Find the magnitude of the resultant vector.

11. For these vectors: $\vec{v}_1 = (5,45^\circ), \vec{v}_2 = (0,-10),$ and $\vec{v}_3 = (1,180^\circ)$
    a. Add the vectors.
    b. Find the magnitude of the resultant vector.
Adding Displacement Vectors

A displacement vector is a quantity that contains two separate pieces of information: (1) magnitude or size, and (2) direction. When you add displacement vectors, you end up at a certain position. This new position is the total displacement from the original position. A vector that connects the starting position with the final position is called the resultant vector ($\vec{x}$).

**Example**

Andreas walked 5 meters east away from a tree. Then, he walked 3 meters north. Finally, he walked 1 meter west. Each of these three pathways is a displacement vector. Use these displacement vectors to find Andreas’s total displacement from the tree.

<table>
<thead>
<tr>
<th>Displacement vector</th>
<th>Direction</th>
<th>Magnitude (meters)</th>
<th>Total magnitude (total meters walked)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>east</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td>north</td>
<td>3</td>
<td>$5 + 3 = 8$</td>
</tr>
<tr>
<td>3</td>
<td>west</td>
<td>1</td>
<td>$8 + 1 = 9$</td>
</tr>
</tbody>
</table>

Andreas’s motion can be represented on a graph. To determine his total displacement from the tree, do the following:

1. Add the east and west displacement vectors. These are in the $x$-axis direction on a graph.
   \[
   \text{Andreas’s walk} = 5 \text{ m east} + (-1)\text{ m west} = 4 \text{ m east}
   \]

2. Add the north and south displacement vectors. These are in the $y$-axis direction on a graph.
   \[
   \text{Andreas’s walk} = 3 \text{ m north}
   \]

The total displacement is 4 meters east and 3 meters north.
Andreas walked a total of 9 meters. The resultant vector ($\vec{x}$) goes from the starting position to the final position of total displacement.

**Practice 1**

1. What is the total displacement of a bee that flies 2 meters east, 5 meters north, and 3 meters east?

2. What is the total displacement of an ant that walks 2 meters west, 3 meters south, 4 meters east, and 1 meter north?

3. A ball is kicked 10 meters north, 5 meters west, 15 meters south, 5 meters east, and 5 meters north. Find the total displacement and the total distance it traveled.
Adding displacement vectors using x-y coordinates

A resultant vector can be written using x-y coordinates on a graph. The original position is the origin of a graph where the axes represent east-west and north-south positions. For example, (2,3)m is a resultant vector with the following components: 2 meters east and 3 meters north. A resultant vector, (-3,-1)m, has components 3 meters west and 1 meter south. Use this information to solve the following problems. Write your answers using x-y coordinates.

**EXAMPLE**

Add the following four vectors to find the resultant vector, \( \vec{x}_R \):
\[
\begin{align*}
x_1 &= (5,0)m, \\
x_2 &= (0,-5)m, \\
x_3 &= (3,0)m, \\
x_4 &= (-7,0)m
\end{align*}
\]

Add the east-west components: \( 5 \text{ m east} + 0 \text{ m} + 3 \text{ m east} + (-7) \text{ m west} = 1 \text{ m east} \)

Add the north-south components: \( 0 \text{ m} + (-5) \text{ m south} + 0 \text{ m} + 0 \text{ m} = (-5) \text{ m south} \)

\( \vec{x}_R = (1,-5)m \).

**PRACTICE 2**

1. Add the following three vectors to find the resultant vector, \( \vec{x}_R \):
\[
\begin{align*}
\vec{x}_1 &= (-2,0)m, \\
\vec{x}_2 &= (0,-5)m, \\
\vec{x}_3 &= (3,0)m
\end{align*}
\]

2. Add the following vectors to find the resultant vector. Plot the resultant vector (\( \vec{x}_R \)) on the grid to the right:
\[
\begin{align*}
\vec{x}_1 &= (4,0)m, \\
\vec{x}_2 &= (-1,2)m, \\
\vec{x}_3 &= (0,1)m
\end{align*}
\]

3. Add the following three vectors to find the resultant vector, \( \vec{x}_R \):
\[
\begin{align*}
\vec{x}_1 &= (5,3)m, \\
\vec{x}_2 &= (-5,0)m, \\
\vec{x}_3 &= (5,2)m
\end{align*}
\]

4. Add the following three vectors to find the resultant vector, \( \vec{x}_R \):
\[
\begin{align*}
\vec{x}_1 &= (6,-2)m, \\
\vec{x}_2 &= (-3,1)m, \\
\vec{x}_3 &= (3,3)m
\end{align*}
\]

5. Add the following three vectors to find the resultant vector, \( \vec{x}_R \):
\[
\begin{align*}
\vec{x}_1 &= (4,4)m, \\
\vec{x}_2 &= (-2,-6)m, \\
\vec{x}_3 &= (0,2)m
\end{align*}
\]
**Projectile Motion**

**Projectile motion has vertical and horizontal components**

Projectile motion has vertical and horizontal components. Gravity affects the vertical motion of an object. When we drop a ball from a height, we know that its speed increases as it falls. The increase in vertical speed is due to the acceleration gravity, \( g = 9.8 \text{ m/sec}^2 \). So the vertical speed of the ball will increase by 9.8 m/sec after each second. After the first second has passed, the speed will be 9.8 m/sec. After the next second has passed, the speed will be 19.6 m/sec and so on.

The acceleration of gravity affects only the vertical component of the motion. Horizontal motion is not affected by gravity. So if we neglect the friction from air, when we throw an object horizontally, its initial horizontal speed will not change. For example, if we throw a marble horizontally at a speed of 5 m/sec, the marble will be 5 meters horizontally from our hand after one second, 10 meters after 2 seconds, and so forth.

**Solving projectile motion problems**

Solving projectile motion problems requires using equations. To solve these problems, follow the steps:

- Read the problem carefully. You may want to diagram the problem to help you understand it.
- List what you know from the problem and what you need to solve for.
- Determine which equations for vertical motion or horizontal motion will help you solve the problem. You may need more than one equation to solve the problem. Some important equations are listed below.
- Solve the problem and check your work.

<table>
<thead>
<tr>
<th>Equation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( d_x = v_x t )</td>
<td>Horizontal distance. This equation is a rearranged version of the speed equation: ( v = d / t ). ( v_x ) should be read as “the initial velocity in the ( x )-direction.”</td>
</tr>
<tr>
<td>( v_y = gt )</td>
<td>Vertical velocity. Gravity (( g )) is included in these equations because vertical speed accelerates due to gravity when an object is falling.</td>
</tr>
<tr>
<td>( d_y = 4.9t^2 )</td>
<td>Vertical distance</td>
</tr>
</tbody>
</table>

- The equations above are suitable ONLY for situations where the projectile starts with zero vertical velocity, like a ball rolling off a table. If the projectile is launched up or down at an angle, the equations are more complicated.
A boy runs at a speed of 3.3 meters per second straight off the end of a diving board that is 3 meters above the water. How long is he airborne before he hits the water? What is the horizontal distance he travels while airborne?

a. What do you know?  
   speed and height  

b. What do you need to solve for?  
   time and horizontal distance  

c. What equations will you use?  
   \[ d_y = 4.9t^2 \quad d_x = v_x t \]

What is the solution to this problem?

\[
\begin{align*}
\text{time:} & \quad \text{horizontal distance:} \\
3.0 \text{ meters} &= 4.9 \quad t^2 & d_x &= 3.3 \text{ m/sec} \times 0.78 \text{ sec} \\
0.61 &= t^2 & d_x &= 2.6 \text{ meters} \\
\sqrt{0.61} &= t & 0.78 \text{ sec} &= t
\end{align*}
\]

Solve the following problems. Show your work.

1. A cat runs and jumps from one rooftop to another which is 5 meters away and 3 meters below. Calculate the minimum horizontal speed with which the cat must jump off the first rooftop in order to make it to the other.

   a. What do you know?  
   b. What do you need to solve for?  
   c. What equation(s) will you use?  
   d. What is the solution to this problem?

2. An object is thrown off a cliff with a horizontal speed of 10 m/sec. After 3 seconds the object hits the ground. Find the height of the cliff and the total horizontal distance traveled by the object.

   a. What do you know?  
   b. What do you need to solve for?  
   c. What equation(s) will you use?  
   d. What is the solution to this problem?
3. If a marble is released from a height of 10 meters how long would it take for it to hit the ground?
   a. What do you know?
   b. What do you need to solve for?
   c. What equation(s) will you use?
   d. What is the solution to this problem?

4. A ski jumper competing for an Olympic gold medal wants to jump a horizontal distance of 135 meters. The takeoff point of the ski jump is at a height of 25 meters. With what horizontal speed must he leave the jump?
   a. What do you know?
   b. What do you need to solve for?
   c. What equation(s) will you use?
   d. What is the solution to this problem?

5. A motorcycle stunt driver zooms off the end of a cliff at a speed of 30 meters per second. If he lands after 0.75 seconds, what is the height of the cliff?
   a. What do you know?
   b. What do you need to solve for?
   c. What equation(s) will you use?
   d. What is the solution to this problem?

6. A marble rolling at a speed of 2 meters per second falls off the end of a 1-meter high table. How long will the marble be in the air?
   a. What do you know?
   b. What do you need to solve for?
   c. What equation(s) will you use?
   d. What is the solution to this problem?

7. **Challenge!** A marshmallow is dropped from a 5-meter high pedestrian bridge and 0.83 seconds later, it lands right on the head of an unsuspecting person walking underneath. How tall is the person with the marshmallow on his head?
   a. What do you know?
   b. What do you need to solve for?
   c. What equation(s) will you use?
   d. What is the solution to this problem?
Circular Motion

You have learned several important terms used to describe circular motion:

- **Rotate** means to spin around an internal axis. Example: Earth makes one complete rotation every 24 hours.
- **Revolve** means to travel in a circle around an external axis. Example: Earth makes one complete revolution around the sun each year.
- **Angular speed** describes how fast something rotates. Degrees per minute and rotations per minute (rpm) are two common units of angular speed.

\[
\text{Angular speed} = \frac{\text{rotations or degrees}}{\text{time}}
\]

- The **radius** is the distance from the axis of rotation to any point on the outside of the circle.
- **Circumference** describes the distance traveled during one revolution.

\[
\text{Circumference} = 2\pi r, \text{ where } r \text{ is the radius of the circle.}
\]

- **Linear speed** describes how fast a revolving object travels. Linear speed is often given in meters per second.

\[
\text{Linear speed} (v) = \frac{2\pi r}{t}, \text{ where } r \text{ is the radius and } t \text{ is the time for one revolution.}
\]

**EXAMPLES**

1. A merry-go-round makes 18 rotations in 3 minutes. What is its angular speed in rpm?

   \[
   \text{Angular speed} = \frac{18 \text{ rotations}}{3 \text{ minutes}} = 6 \text{ rpm}
   \]

2. A coin rolls across the floor at an angular speed of 4 rotations per second. What is its speed in degrees per second? Hint: One full rotation equals 360 degrees.

   \[
   \text{Angular speed} = \frac{4 \times 360^\circ}{1 \text{ second}} = 1440^\circ/\text{second}
   \]

3. A child sits two meters from the center of a merry-go-round. How far does she travel during one revolution?

   \[
   \text{Circumference} = 2\pi(2 \text{ meters}) = 12.6 \text{ meters}
   \]

4. If the merry-go-round makes one revolution in 10 seconds, what is the child’s linear speed?

   \[
   \text{Linear speed} = \frac{2\pi(2 \text{ meters})}{10 \text{ seconds}} = 1.3 \text{ m/sec}
   \]
1. A compact disc is spinning with an angular speed of 3.3 rotations per second.
   a. What is its angular speed in degrees per second?
   b. What is its angular speed in rotations per minute (rpm)?

2. A compact disc has a radius of 6 centimeters.
   a. What is its circumference in meters?
   b. If the cd rotates 4 times per second, what is the linear speed of a point on the outer edge of the cd? Give your answer in meters per second.
   c. What is the linear speed of a point 3 centimeters from the center of the cd? (Assume the angular speed has not changed).

3. **Challenge!** When a computer reads a cd-rom, the “read-head” must read the data at a constant linear velocity. That means the same amount of information must pass by the “read-head” each second no matter what part of the cd is being read. The cd spins at different angular speeds to keep the linear speed the same. If the “read-head” moves from reading data at the inner edge of the cd to read data at the outer edge, will the cd need to spin faster or slower to maintain a constant linear velocity?

4. Rolling is a combination of linear and rotating motion. When a wheel makes one full rotation, it moves forward a distance equal to the wheel’s circumference.
   a. A child’s first bicycle has 12-inch tires. These tires have a 6-inch radius. How far does the bicycle move forward each time the wheel makes one complete rotation? Give your answer in meters.
      (1 inch = 0.022 meters)
   b. A woman’s ten-speed bicycle has 27-inch tires (13.5-inch radius). How far does this bicycle move forward each time the wheel makes one complete rotation? Give your answer in meters.
   c. How many times does the child’s bicycle tire have to rotate for the bicycle to travel 1 kilometer?
   d. How many times does the woman’s bicycle tire have to rotate for the bicycle to travel 1 kilometer?
Universal Gravitation

The law of universal gravitation allows you to calculate the gravitational force between two objects from their masses and the distance between them. The law includes a value called the gravitational constant, or “G.” This value is the same everywhere in the universe. Calculating the force between small objects like grapefruits or huge objects like planets, moons, and stars is possible using this law.

What is the law of universal gravitation?

The force between two masses \( m_1 \) and \( m_2 \) that are separated by a distance \( r \) is given by:

\[
F = G \frac{m_1 m_2}{r^2}
\]

So, when the masses \( m_1 \) and \( m_2 \) are given in kilograms and the distance \( r \) is given in meters, the force has the unit of newtons. Remember that the distance \( r \) corresponds to the distance between the center of gravity of the two objects.

For example, the gravitational force between two spheres that are touching each other, each with a radius of 0.3 meter and a mass of 1,000 kilograms, is given by:

\[
F = 6.67 \times 10^{-11} \text{ N-m}^2/\text{kg}^2 \frac{1,000 \text{ kg} \times 1,000 \text{ kg}}{(0.3 \text{ m} + 0.3 \text{ m})^2} = 0.000185 \text{ N}
\]

Note: A small car has a mass of approximately 1,000 kilograms. Try to visualize this much mass compressed into a sphere with a diameter of 0.3 meters (30 centimeters). If two such spheres were touching one another, the gravitational force between them would be only 0.000185 newtons. On Earth, this corresponds to the weight of a mass equal to only 18.9 milligrams. The gravitational force is not very strong!
Answer the following problems. Write your answers using scientific notation.

1. Calculate the force between two objects that have masses of 70 kilograms and 2,000 kilograms separated by a distance of 1 meter.

2. Calculate the force between two touching grapefruits each with a radius of 0.08 meters and a mass of 0.45 kilograms.

3. Calculate the force between one grapefruit as described above and Earth. Earth has a mass of $5.9742 \times 10^{24}$ kg and a radius of $6.3710 \times 10^6$ meters. Assume the grapefruit is resting on Earth’s surface.

4. A man on the moon with a mass of 90 kilograms weighs 146 newtons. The radius of the moon is $1.74 \times 10^6$ meters. Find the mass of the moon.

5. For $m = 5.9742 \times 10^{24}$ kilograms and $r = 6.378 \times 10^6$ meters, what is the value given by this equation: $G \frac{m^2}{r^2}$?
   a. Write down your answer and simplify the units.
   b. What does this number remind you of?
   c. What real-life values do $m$ and $r$ correspond to?

6. The distance between Earth and its moon is $3.84 \times 10^8$ meters. Earth’s mass is $m = 5.9742 \times 10^{24}$ kilograms and the mass of the moon is $7.36 \times 10^{22}$ kilograms. What is the force between Earth and the moon?

7. A satellite is orbiting Earth at a distance of 35 kilometers. The satellite has a mass of 500 kilograms. What is the force between the planet and the satellite?

8. The mass of the sun is $1.99 \times 10^{30}$ kilograms and its distance from Earth is 150 million kilometers ($150 \times 10^9$ meters). What is the gravitational force between the sun and Earth?
Indirect Measurement

Have you ever wondered how scientists and engineers measure large quantities like the mass of an iceberg, the volume of a lake, or the distance across a river? Obviously, balances, graduated cylinders, and measuring tapes could not do the job! Very large (or very small) quantities are calculated through a process called indirect measurement. This skill sheet will give you an opportunity to try indirect measurement for yourself.

**EXAMPLES**

• The length of a tree’s shadow is 4.25 meters and the length of a meter stick’s shadow is 1.25 meters. Using these two values and the length of the meter stick, how tall is the tree?

<table>
<thead>
<tr>
<th>Looking for</th>
<th>Solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>The height of a tree.</td>
<td></td>
</tr>
<tr>
<td><strong>Given</strong></td>
<td></td>
</tr>
<tr>
<td>Tree’s shadow = 4.25 m</td>
<td>1.00 m</td>
</tr>
<tr>
<td>Meter’s stick’s shadow = 1.23 m</td>
<td>1.25 m</td>
</tr>
<tr>
<td>Height of meter stick = 1 m</td>
<td>4.25 m</td>
</tr>
<tr>
<td><strong>Relationships</strong></td>
<td></td>
</tr>
<tr>
<td>There is a direct relationship between the height of objects and the length of their shadows.</td>
<td></td>
</tr>
</tbody>
</table>
| \[
\frac{\text{height of meter stick}}{\text{length of meter stick shadow}} = \frac{\text{height of object}}{\text{length of object shadow}}
\] |          |

\[
4.25 m \times \frac{1.00 m}{1.25 m} = \frac{\text{height of tree}}{4.25 m} \times 4.25 m
\]

\[
3.40 \text{ m} = \text{height of tree}
\]

The height of the tree is 3.40 meters.

• At the science museum, 12 first graders stand on a giant scale to measure their mass. The combined mass of the 12 first graders is 262 kilograms. What is the average mass of a first grader in this group?

<table>
<thead>
<tr>
<th>Looking for</th>
<th>Solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>The average mass of a first grader.</td>
<td></td>
</tr>
<tr>
<td><strong>Given</strong></td>
<td></td>
</tr>
<tr>
<td>Total mass of 12 first graders = 262 kilograms</td>
<td></td>
</tr>
<tr>
<td><strong>Relationships</strong></td>
<td></td>
</tr>
<tr>
<td>To get the average mass of one first grader, divide the total mass by 12.</td>
<td></td>
</tr>
</tbody>
</table>
| \[
\frac{262 \text{ kilograms}}{12} = \frac{262}{12} = 21.8 \text{ kilograms}
\] |          |

The average mass of a first grader in this group is 21.8 kilograms.

**PRACTICE**

1. You go to another forest and measure the shadow of a tree as being 6 meters long. The shadow of your meter stick is 2 meters long. How tall is the tree?

2. The height of a flagpole is 8.5 meters tall. If the length of the meter stick shadow is 1.5 meters long, what would be the length of the flagpole’s shadow?
3. While touring a city, you see a skyscraper and wonder how tall it is. You see that it is clearly divided into floors. You estimate that each floor is 20 feet high. You count that the skyscraper has 112 floors.
   a. How tall is this skyscraper in feet?
   b. How tall is the skyscraper in meters? (One foot is about 0.3 meter.)

4. There are 10 apples in a one-kilogram bag of apples. What is the average mass of each apple in the bag? Give your answer in units of kilograms and grams.

5. If you place one staple on an electronic balance, the balance still reads 0.0 grams. However, if you place 210 staples on the balance, it reads 6.80 grams. What is the mass of one staple?

6. A stack of 55 business cards is 1.85 cm tall. Use this information to determine the thickness of one business card.

7. A stack of eight compact disks is 1.0 centimeter high. What is the thickness of one compact disk (CD) in centimeters? What is the thickness of one CD in millimeters?

8. A quarter is 2.4 millimeters thick. How tall are the following stacks of quarters?
   a. A stack worth 50 cents
   b. A stack worth $1
   c. A stack worth $5
   d. A stack worth $1,000
      (Give your answer in millimeters and meters.)

9. Yvonne has gained a reputation for her cheesecakes. She takes orders for 10 cheesecakes and spends 8.5 hours on one Saturday baking them all. She earns $120 from selling these cheesecakes.
   a. What is the average length of time to make one cheesecake? Give your answer in minutes.
   b. What does Yvonne charge for each cheesecake?
   c. How much money is Yvonne earning per hour for her work?

10. A sculptor wants to create a statue. She goes to a quarry to buy a block of marble. She finds a chip of marble on the ground. The volume of the chip is 15.3 cm$^3$. The mass of the chip is 41.3 grams. The sculptor purchases a block of marble 30.0-by-40.0-by-100.0 cm. Use a proportion to find the mass of her block of marble.

11. The instructions on a bottle of eye drops say to place three drops in each eye, using the dropper. How could you find the volume of one of these drop? Write a procedure for finding the volume of a drop that includes using a glass of water, a 10.0-mL graduated cylinder, and the dropper.

12. A student wants to use indirect measurement to find the thickness of a sheet of newspaper. In a 50-centimeter tall recycling bin, she finds 50 sheets of newspaper. Each sheet in the bin is folded in fourths. Design a procedure for the student to use that would allow her to measure the thickness of one sheet of newspaper with little or no source of experimental error. The student has a meter stick and a calculator.
## Temperature Scales

The Fahrenheit and Celsius temperature scales are the most commonly used scales for reporting temperature values. Scientists use the Celsius scale almost exclusively, as do many countries of the world. The United States relies on the Fahrenheit scale for reporting temperature information. You can convert information reported in degrees Celsius to degrees Fahrenheit or vice versa using conversion formulas.

**Fahrenheit (°F) to Celsius (°C) conversion formula:**

\[ °C = \frac{5}{9}(°F - 32) \]

**Celsius (°C) to Fahrenheit (°F) conversion formula:**

\[ °F = \left(\frac{9}{5} \times °C\right) + 32 \]

### EXAMPLE

- What is the Celsius value for 65° Fahrenheit?

\[ °C = \frac{5}{9}(65° - 32) \]

\[ °C = 18.3 \]

- 200°C is the same temperature as what value on the Fahrenheit scale?

\[ °F = \frac{9}{5}(200°C) + 32 \]

\[ °F = 392 \]

### PRACTICE 1

1. The weatherman reports that today will reach a high of 45°F. Your friend from Sweden asks what the temperature will be in degrees Celsius. What value would you report to your friend?

2. Your parents order an oven from England. The temperature dial on the new oven is calibrated in degrees Celsius. If you need to bake a cake at 350°F in the new oven, at what temperature should you set the dial?

3. A German automobile's engine temperature gauge reads in Celsius, not Fahrenheit. The engine temperature should not rise above about 225°F. What is the corresponding Celsius temperature on this car's gauge?

4. Your grandmother in Ireland sends you her favorite cookie recipe. Her instructions say to bake the cookies at 190.5°C. To what Fahrenheit temperature would you set the oven to bake the cookies?

5. A scientist wishes to generate a chemical reaction in his laboratory. The temperature values in his laboratory manual are given in degrees Celsius. However, his lab thermometers are calibrated in degrees Fahrenheit. If he needs to heat his reactants to 232°C, what temperature will he need to monitor on his lab thermometers?

6. You call a friend in Denmark during the Christmas holidays and say that the temperature in Boston is 15 degrees. He replies that you must enjoy the warm weather. Explain his comment using your knowledge of the Fahrenheit and Celsius scales. To help you get started, fill in this table. What is 15°F on the Celsius scale? What is 15°C on the Fahrenheit scale?

<table>
<thead>
<tr>
<th>°F</th>
<th>°C</th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td></td>
</tr>
<tr>
<td></td>
<td>15°C</td>
</tr>
</tbody>
</table>

### Fill in the Table

\[ °C = \frac{5}{9}(15°F - 32) \]

\[ °C = 6.11 \]

\[ °F = \frac{9}{5} × 15°C + 32 \]

\[ °F = 59°F \]
Extension: the Kelvin temperature scale

For some scientific applications, a third temperature scale is used: the Kelvin scale. On the Kelvin scale, 0 K (degree symbols are not used for Kelvin values) represents absolute zero. Absolute zero is equal to -273°C, or -459°F. When scientists are conducting research, they often obtain or report their temperature values in Celsius, and other scientists must convert these values into Kelvin for their own use, or vice versa. To convert Celsius values to their Kelvin equivalents, you would use the formula:

\[ K = ^\circ C + 273 \]

**EXAMPLE**

- Water boils at a temperature of 100°C. What would be the corresponding temperature for the Kelvin scale?

\[ K = ^\circ C + 273 \]

\[ K = 100^\circ C + 273 = 373 \]

To convert Kelvin values to Celsius, you would perform the opposite operation; subtract 273 from the Kelvin value to find the Celsius equivalent.

- A substance has a melting point of 625 K. At what Celsius temperature would this substance melt?

\[ ^\circ C = K - 273 \]

\[ ^\circ C = 625 K - 273 = 352 \]

Although we rarely need to convert between Kelvin and Fahrenheit, use the following formulas to do so:

\[ K = \frac{5}{9}(^\circ F + 460) \]

\[ ^\circ F = \left(\frac{9}{5} \times K\right) - 460 \]

**PRACTICE 2**

1. A gas has a boiling point of -175°C. At what Kelvin temperature would this gas boil?

2. A chemist notices some silvery liquid on the floor in her lab. She wonders if someone accidentally broke a mercury thermometer, but did not thoroughly clean up the mess. She decides to find out of the silver stuff is really mercury. From her tests with the substance, she finds out that the melting point for the liquid is 275 K. A reference book says that the melting point for mercury is -38.87°C. Is this substance mercury? Explain your answer and show all relevant calculations.

3. It is August 1st and you are at a Science Camp in Florida. During an outdoor science quiz, you are asked to identify the temperature scale for a thermometer that reports the current temperature as 90. Is this thermometer calibrated for the Kelvin, Fahrenheit, or Celsius temperature scale? Fill in the table below to answer this question.

<table>
<thead>
<tr>
<th>°F</th>
<th>°C</th>
<th>K</th>
</tr>
</thead>
<tbody>
<tr>
<td>90°F</td>
<td>=</td>
<td>=</td>
</tr>
<tr>
<td>=</td>
<td>90°C</td>
<td>=</td>
</tr>
<tr>
<td>=</td>
<td>=</td>
<td>90</td>
</tr>
</tbody>
</table>
Specific Heat

Specific heat is defined as the amount of heat energy needed to raise 1 gram of a substance 1°C in temperature.

- Specific heat values are used in the heat equation is:
  
  \[ Q = mC_p(T_2 - T_1) \]

  where \( Q \) is the heat energy (joules), \( m \) is the mass of the substance (kilograms), \( C_p \) is the specific heat of the substance (J/kg°C), and \((T_2 - T_1)\) is the change in temperature (°C)

- The higher the specific heat, the more energy is required to cause a change in temperature. Substances with higher specific heats require more loss of heat energy to experience a lowering of their temperature than do substances with a low specific heat. Some sample specific heat values are presented in the table below:

<table>
<thead>
<tr>
<th>Material</th>
<th>Specific Heat (J/kg °C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>water (pure)</td>
<td>4,184</td>
</tr>
<tr>
<td>aluminum</td>
<td>900</td>
</tr>
<tr>
<td>steel</td>
<td>470</td>
</tr>
<tr>
<td>silver</td>
<td>235</td>
</tr>
<tr>
<td>oil</td>
<td>1,900</td>
</tr>
<tr>
<td>concrete</td>
<td>880</td>
</tr>
<tr>
<td>glass</td>
<td>800</td>
</tr>
<tr>
<td>gold</td>
<td>129</td>
</tr>
<tr>
<td>wood</td>
<td>2,500</td>
</tr>
</tbody>
</table>

- Water has the highest specific heat of the listed types of matter. This means that water is slower to heat but is also slower to lose heat.

**EXAMPLE**

- How much energy is required to heat 35 grams of gold from 10°C to 50°C?

| Looking for | | Solution |
|-------------| |----------|
| The heat energy in joules to heat 35 grams of gold by 40°C. | | \[ Q = mC_p(T_2 - T_1) \] |
| Given       | | \[ Q = (0.35 \text{ kg})(129 \frac{J}{\text{kg} \cdot ^\circ \text{C}})(50^\circ \text{C} - 10^\circ \text{C}) \] |
| Mass = 35 grams = 0.35 kilogram | | \[ Q = (0.35 \text{ kg})(129 \frac{J}{\text{kg} \cdot ^\circ \text{C}})(40^\circ \text{C}) \] |
| Specific heat of gold = 129 J/g°C | | \[ Q = 1,806 \text{ joules} \] |
| \( T_2 = 50^\circ \text{C} \) and \( T_1 = 10^\circ \text{C} \) | | To produce the necessary change in temperature, 1,806 joules of heat energy need to be put into this sample of gold. |
Using the heat formula and the table of specific heat values, solve the following heat problems.

1. A 0.5-kilogram piece of aluminum increases its temperature 7°C when heat energy is added. How much heat energy produced this change in temperature?

2. A volume of water has a mass of 0.5 kilogram. If the temperature of this amount of water was raised by 7°C, how much heat energy is produced?

3. How much heat energy is required to raise the temperature of 1 kilogram of steel by 10°C?

4. How much heat energy is needed to raise the temperature of 100-liters of water from 10°C to 25°C? Note: One liter of water has a mass of one kilogram.

5. When 1,500 joules of energy is lost from a 0.12-kilogram object, the temperature decreases from 45°C to 40°C. What is the specific heat of this object? Of what material is the object made?

6. What is the specific heat of a material that gains 600 joules of energy when a 0.25-kilogram object increases in temperate by 3°C? What is this material?

7. A liquid with a specific heat of 1,900 J/kg·°C has 4,750 joules of heat energy is added to it. If the temperature increases from 20°C to 30°C, what is the mass of the liquid?

8. What is the mass of a block of concrete that gains 52,800 joules of energy when its temperature is increased by 5°C?

9. A scientist wants to raise the temperature of a 0.10-kilogram sample of glass from –45°C to 15°C. How much heat energy is required to produce this change in temperature?

10. A person wishes to heat pot of fresh water from 20°C to 100°C in order to boil water for pasta. They calculate that their pot holds 2 kilograms of water and that they would need to apply 669,440 joules of heat energy to produce the desired temperature change. Are the person’s calculations correct? Defend your answer and demonstrate all relevant calculations.

11. A 0.25-kilogram sample of aluminum is provided with 5,000 joules of heat energy. What will be the change in temperature of this sample of aluminum?

12. What is the change in temperature for a 2-kilogram mass of water that loses 8,500 joules of energy?

13. Which of the substances listed in the table on the first page would heat up more quickly if an equal amount of heat energy were applied to all of the substances at the same time? Explain your answer.

14. Which of the substances listed in the table on the first page would you choose as the best insulator (substance that requires a lot of heat energy to experience a change in temperature)? Explain your answer.

15. Which substance—wood or steel—is the better conductor? A conductor is a material that requires very little heat energy to experience a change in temperature. Explain your answer.
Density

- The density of a substance does not depend on its size or shape. As long as a substance is homogeneous, the density will be the same. This means that a steel nail has the same density as a cube of steel or a steel girder used to build a bridge.

- The formula for density is: $\text{density} = \frac{\text{mass}}{\text{volume}}$

- One milliliter takes up the same amount of space as one cubic centimeter. Therefore, density can be expressed in units of g/mL or g/cm³. Liquid volumes are most commonly expressed in milliliters, while volumes of solids are usually expressed in cubic centimeters.

- Density can also be expressed in units of kilograms per cubic meter (kg/m³).

- If you know the density of a substance and the volume of a sample, you can calculate the mass of the sample. To do this, rearrange the equation above to find mass: $\text{volume} \times \text{density} = \text{mass}$

- If you know the density of a substance and the mass of a sample, you can find the volume of the sample. This time, you will rearrange the density equation to find volume: $\text{volume} = \frac{\text{mass}}{\text{density}}$

### EXAMPLES

**Example 1:** What is the density of a block of aluminum with a volume of 30.0 cm³ and a mass of 81.0 grams?

$$\text{density} = \frac{81.0 \text{ g}}{30.0 \text{ cm}^3} = \frac{2.70 \text{ g}}{\text{cm}^3}$$

**Answer:** The density of aluminum is 2.70 g/cm³.

**Example 2:** What is the mass of an iron horseshoe with a volume of 89 cm³? The density of iron is 7.9 g/cm³.

$$\text{mass} = 89 \text{ cm}^3 \times 7.9 \frac{\text{g}}{\text{cm}^3} = 703 \text{ grams}$$

**Answer:** The mass of the horseshoe is 703 grams.

**Example 3:** What is the volume of a 525-gram block of lead? The density of lead is 11.3 g/cm³.

$$\text{volume} = \frac{525 \text{ g}}{11.3 \frac{\text{g}}{\text{cm}^3}} = 46.5 \text{ cm}^3$$

**Answer:** The volume of the block is 46.5 cm³.
8.1

1. A solid rubber stopper has a mass of 33.0 grams and a volume of 30.0 cm³. What is the density of rubber?

2. A chunk of paraffin (wax) has a mass of 50.4 grams and a volume of 57.9 cm³. What is the density of paraffin?

3. A marble statue has a mass of 6,200 grams and a volume of 2,296 cm³. What is the density of marble?

4. The density of ice is 0.92 g/cm³. An ice sculptor orders a one cubic meter block of ice. What is the mass of the block? Hint: 1 m³ = 1,000,000 cm³. Give your answer in grams and kilograms.

5. What is the mass of a pure platinum disk with a volume of 113 cm³? The density of platinum is 21.4 g/cm³. Give your answer in grams and kilograms.

6. The density of seawater is 1.025 g/mL. What is the mass of 1.000 liter of seawater in grams and in kilograms? (Hint: 1 liter = 1,000 mL)

7. The density of cork is 0.24 g/cm³. What is the volume of a 240-gram piece of cork?

8. The density of gold is 19.3 g/cm³. What is the volume of a 575-gram bar of pure gold?

9. The density of mercury is 13.6 g/mL. What is the volume of a 155-gram sample of mercury?

10. Recycling centers, for example, use density to help sort and identify different types of plastics so that they can be properly recycled. The table below shows common types of plastics, their recycling code, and density. Use the table to solve problems 10a -d.

<table>
<thead>
<tr>
<th>Plastic name</th>
<th>Common uses</th>
<th>Recycling code</th>
<th>Density (g/cm³)</th>
<th>Density (kg/m³)</th>
</tr>
</thead>
<tbody>
<tr>
<td>PETE</td>
<td>plastic soda bottles</td>
<td>1</td>
<td>1.38-1.39</td>
<td>1,380 - 1,390</td>
</tr>
<tr>
<td>HDPE</td>
<td>milk cartons</td>
<td>2</td>
<td>0.95-0.97</td>
<td>950 - 970</td>
</tr>
<tr>
<td>PVC</td>
<td>plumbing pipe</td>
<td>3</td>
<td>1.15-1.35</td>
<td>1,150 - 1,350</td>
</tr>
<tr>
<td>LDPE</td>
<td>trash can liners</td>
<td>4</td>
<td>0.92-0.94</td>
<td>920 - 940</td>
</tr>
<tr>
<td>PP</td>
<td>yogurt containers</td>
<td>5</td>
<td>0.90-0.91</td>
<td>900 - 910</td>
</tr>
<tr>
<td>PS</td>
<td>cd “jewel cases”</td>
<td>6</td>
<td>1.05-1.07</td>
<td>1,050 - 1,070</td>
</tr>
</tbody>
</table>

a. A recycling center has a 0.125 m³ box filled with one type of plastic. When empty, the box had a mass of 0.755 kilograms. The full box has a mass of 120.8 kilograms. What is the density of the plastic? What type of plastic is in the box?

b. A truckload of plastic soda bottles was finely shredded at a recycling center. The plastic shreds were placed into 55-liter drums. What is the mass of the plastic shreds inside one of the drums? Hint: 55 liters = 55,000 milliliters = 55,000 cm³.

c. A recycling center has 100 kilograms of shredded plastic yogurt containers. What volume is needed to hold this amount of shredded plastic? How many 10-liter (10,000 mL) containers do they need to hold all of this plastic? Hint: 1 m³ = 1,000,000 mL.

d. A solid will float in a liquid if it is less dense than the liquid, and sink if it is more dense than the liquid. If the density of seawater is 1.025 g/mL, which types of plastics would definitely float in seawater?
Stress

Stress is the ratio of the force acting through the material divided by the cross-section area through which the force is carried. The cross-section area is the area perpendicular to the direction of the force.

\[ \sigma = \frac{F}{A} \]

The metric unit of stress is the pascal (Pa). One pascal is equal to one newton of force per square meter of area (1 N/m²). Most stresses are much larger than one pascal. Strong materials like steel and aluminum can take stresses of 100 million pascals. The English unit for stress is pounds per square inch (psi). A stress of one psi is equivalent to one pound of force for each square inch of area (1 lb/in²).

Tensile strength is a measure of how much stress a material can withstand before breaking.

**EXAMPLE**

A steel beam with a cross-section of 0.5 m² experiences 10,000 newtons of force. What is the stress on the beam?

<table>
<thead>
<tr>
<th>Looking for</th>
<th>Solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>The stress a steel beam.</td>
<td>( \sigma = \frac{F}{A} )</td>
</tr>
<tr>
<td>Given</td>
<td>( \sigma = \frac{10,000 \text{ N}}{0.5 \text{ m}^2} )</td>
</tr>
<tr>
<td></td>
<td>( \sigma = 20,000 \text{ N/m}^2 )</td>
</tr>
<tr>
<td>Relationship</td>
<td>The stress on the beam is 20,000 N/m² or 20,000 pascals.</td>
</tr>
</tbody>
</table>

\( \sigma \) = stress (Pa or N/m²)
\( F \) = force (N)
\( A \) = cross-section area (m²)
1. What is the stress on a steel beam with a 1 m$^2$ cross-section if a force of 15,000 newtons is applied to it?

2. A wooden beam breaks when 30,000,000 newtons of force is applied to it. The cross-section of the beam is 0.5 m$^2$. The tensile strength of pine is 60,000,000 pascals and the tensile strength of oak is 95,000,000. Is this wooden beam made of pine or oak?

3. You have a box of pencils and each one has a cross-section of 0.002 m$^2$. The maximum stress that each pencil can take is 22,500 pascals. How many pencils could be broken at once by a force of 135 newtons?

4. What is the cross-section of an object that experiences a stress of 6,000 N/m$^2$ when a force of 2,400 newtons is applied to it?

5. What is the cross-section of a material that experiences a stress of 500 N/m$^2$ when a force of 50 newtons is applied?
Buoyancy

When an object is placed in a fluid (liquid or gas), the fluid exerts an upward force upon the object. This force is called a **buoyant force**.

At the same time, there is an attractive force between the object and Earth, which we call the force of gravity. It acts as a **downward force**.

### Examples

**Example 1:** A 13-newton object is placed in a container of fluid. If the fluid exerts a 60-newton buoyant (upward) force on the object, will the object float or sink?
**Answer:** Float. The upward buoyant force (60 N) is greater than the weight of the object (13 N).

**Example 2:** The rock weighs 2.25 newtons when suspended in air. In water, it appears to weigh only 1.8 newtons. Why?
**Answer:** The water is exerts a buoyant force on the rock. This buoyant force equals the difference between the rock’s weight in air and its apparent weight in water.

\[
2.25 \text{ N} - 1.8 \text{ N} = 0.45 \text{ N}
\]

The water exerts a buoyant force of 0.45 newtons on the rock.

### Practice

1. A 4.5-newton object is placed in a tank of water. If the water exerts a force of 4.0 newtons on the object, will the object sink or float?
2. The same 4.5-newton object is placed in a tank of glycerin. If the glycerin exerts a force of 5.0 newtons on the object, will the object sink or float?
3. You suspend a brass ring from a spring scale. Its weight is 0.83 N while it is suspended in air. Next, you immerse the ring in a container of light corn syrup. The ring appears to weigh 0.71 N. What is the buoyant force acting on the ring in the light corn syrup?
4. You wash the brass ring (from question 3) and then suspend it in a container of vegetable oil. The ring appears to weigh 0.73 N. What is the buoyant force acting on the ring?
5. Which has greater buoyant force, light corn syrup or the vegetable oil? Why do you think this is so?
6. A cube of gold weighs 1.89 N when suspended in air from a spring scale. When suspended in molasses, it appears to weigh 1.76 N. What is the buoyant force acting on the cube?
7. Do you think the buoyant force would be greater or smaller if the gold cube were suspended in water? Explain your answer.
Archimedes Principle

Have you ever tried to hold a beach ball underwater? It takes a lot of effort! That’s because the buoyant force is much larger than the gravitational force acting on the beach ball.

We can use Archimedes principle to calculate the buoyant force acting on the beach ball. **Archimedes principle** states:

The buoyant force acting on an object in a fluid is equal to the weight of the fluid displaced by the object.

A beach ball has a volume of 14,130 cm³. This means that if you push the ball underwater, it displaces 14,130 cm³ of water. Archimedes principle tells us that the buoyant force on the ball is equal to the weight of that water.

Because the weight of 14,130 cm³ of water is 138 newtons, the buoyant force acting on the beach ball is 138 newtons.

In air, a beach ball weighs 1.5 newtons. However, if you measure the weight of a floating beach ball in water, a spring scale reads 0.0 newtons. The apparent weight of the ball is 0.0 newtons.

The buoyant force acting on the floating beach ball is equal to:

\[( \text{The gravitational force acting on the ball}) - (\text{Apparent weight of ball in water})\]

\[1.5 \text{ N} - 0.0 \text{ N} = 1.5 \text{ N}\]

The buoyant force acting on the floating beach ball is equal to the gravitational force pulling the ball downward.

The floating ball displaces only 153 cm³ of water. 153 cm³ of water weighs 1.5 newtons. The ball displaces an amount of water equal to its own weight.

**EXAMPLE**

A 5-cm³ block of lead weighs 0.55 N. The lead is carefully submerged in a tank of mercury. One cm³ of mercury weighs 0.13 N. What is the weight of the mercury displaced by the block of lead? Will the block of lead sink or float in the mercury?

<table>
<thead>
<tr>
<th>Given</th>
<th>Solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>Volume of block = 5 cm³</td>
<td>The lead will displace 5 cm³ of mercury.</td>
</tr>
<tr>
<td>Weight of lead block = 0.55 N</td>
<td>5 cm³ mercury × 0.13 N/1 cm³ mercury = 0.65 N</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Looking for</th>
<th>The buoyant force of mercury, 0.65 N, is greater than the weight of the lead, 0.55 N. Therefore, the block of lead will float.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weight of mercury displaced</td>
<td></td>
</tr>
<tr>
<td>Will the lead sink or float?</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Relationships</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 cm³ mercury weighs 0.13 N</td>
</tr>
</tbody>
</table>
1. A 10 cm$^3$ block of paraffin (a type of wax) weighs 0.085 N. It is carefully submerged in a container of gasoline. One cm$^3$ of gasoline weighs 0.0069 N.
   a. What is the weight of the gasoline displaced by the paraffin?
   b. Will the block of paraffin sink or float in the gasoline?

2. A 30 cm$^3$ chunk of platinum weighs 6.3 N. It is carefully submerged in a tub of molasses. One cm$^3$ of molasses weighs 0.013 N.
   a. What is the weight of the molasses displaced by the platinum?
   b. Will the platinum sink or float in the molasses?

3. A 15 cm$^3$ block of gold weighs 2.8 N. It is carefully submerged in a tank of mercury. One cm$^3$ of mercury weighs 0.13 N.
   a. Will the mercury be displaced by the gold?
   b. Will the gold sink or float in the mercury?

4. Compare the densities of each pair of materials in questions 1-3 above.
   a. paraffin versus gasoline
   b. platinum versus molasses
   c. gold versus mercury

5. Does an object’s density have anything to do with whether or not it will float in a particular liquid? Justify your answer.

6. Based on density, explain whether the object would float or sink in the following situations:
   a. A block of solid paraffin (wax) in molasses.
   b. A gold ring in molten platinum.
   c. A piece of platinum in molten gold.
   d. A drop of gasoline in mercury.
   e. A drop of mercury in gasoline.
The relationship between the volume of a gas and the pressure of a gas, at a constant temperature, is known as Boyle’s law. The equation for Boyle’s law is at right.

Units for pressure include: atmospheres (atm), pascals (Pa), or kilopascals (kPa). Units for volume include: cubic centimeters (cm³), cubic meters (m³), or liters.

A kit used to fix flat tires consists of an aerosol can containing compressed air and a patch to seal the hole in the tire. Suppose 10.0 liters of air at atmospheric pressure (101.3 kilopascals, or kPa) is compressed into a 1.0-liter aerosol can. What is the pressure of the compressed air in the can?

<table>
<thead>
<tr>
<th>Looking for</th>
<th>Solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pressure of compressed air in a can (P₂)</td>
<td>P₂ = \frac{101.3 \text{ kPa} \times 10.0 \text{ L}}{1.0 \text{ L}} = 1,013 \text{ kPa}</td>
</tr>
</tbody>
</table>

The pressure inside the aerosol can is 1,013 kPa.
1. The air inside a tire pump occupies a volume of 130.0 cubic centimeters at a pressure of one atmosphere. If the volume decreases to 40.0 cubic centimeters, what is the pressure, in atmospheres, inside the pump?

2. A gas occupies a volume of 20 cubic meters at 9,000 pascals. If the pressure is lowered to 5,000 pascals, what volume will the gas occupy?

3. You pump 25.0 liters of air at atmospheric pressure (101.3 kPa) into a soccer ball that has a volume of 4.5 liters. What is the pressure inside the soccer ball if the temperature does not change?

4. Hyperbaric oxygen chambers (HBO) are used to treat divers with decompression sickness. As pressure increases inside the HBO, more oxygen is forced into the bloodstream of the patient inside the chamber. To work properly, the pressure inside the chamber should be three times greater than atmospheric pressure (101.3 kPa). What volume of oxygen, held at atmospheric pressure, will need to be pumped into a 190-liter HBO chamber to make the pressure inside three times greater than atmospheric pressure?

5. A 12.5-liter scuba tank holds of oxygen at a pressure of 202.6 kPa. What is the original volume of oxygen at 101.3 kPa that is required to fill the scuba tank?
The pressure-temperature relationship shows a direct relationship between the pressure of a gas and its temperature when the temperature is given in the Kelvin scale. Another name for this relationship is the Gay-Lussac Law. The pressure-temperature equation is below.

Converting from degrees Celsius to Kelvin is easy — you add 273 to the Celsius temperature. To convert from Kelvins to degrees Celsius, you subtract 273 from the Kelvin temperature.

**Example**

A constant volume of gas is heated from 25.0°C to 100°C. If the gas pressure starts at 1.00 atmosphere, what is the final pressure of this gas?

<table>
<thead>
<tr>
<th>Looking for</th>
<th>Solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>The new pressure of the gas ($P_2$)</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Given</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_1 = 25^\circ C; P_1 = 1$ atm; $T_2 = 100^\circ C$</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Relationships</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Use pressure-temperature relation to solve for $P_2$. Multiply each side by $T_2$ to isolate $P_2$ on one side of the equation.</td>
<td></td>
</tr>
</tbody>
</table>
\[
P_2 = \frac{P_1 T_2}{T_1}
\]

Convert temperature values in Celsius degrees to Kelvin:

\[
T_{\text{Kelvin}} = T_{\text{Celsius}} + 273
\]

\[
P_2 = \frac{1 \text{ atm} \times 373}{298} = 1.25 \text{ atm}
\]

The new pressure of the volume of gas is 1.25 atmospheres.

**Practice**

1. At 400 K, a volume of gas has a pressure of 0.40 atmospheres. What is the pressure of this gas at 273 K?

2. What is the temperature of the volume of gas (starting at 400 K with a pressure of 0.4 atmospheres), when the pressure increases to 1 atmosphere?

3. Use the pressure-temperature relationship to fill in the following table with the correct values. Pay attention to the temperature units.

<table>
<thead>
<tr>
<th></th>
<th>$P_1$</th>
<th>$T_1$</th>
<th>$P_2$</th>
<th>$T_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>a.</td>
<td>30.0 atm</td>
<td>-100°C</td>
<td></td>
<td></td>
</tr>
<tr>
<td>b.</td>
<td>15.0 atm</td>
<td>25.0°C</td>
<td>18.0 atm</td>
<td>500°C</td>
</tr>
<tr>
<td>c.</td>
<td>5.00 atm</td>
<td>3.00 atm</td>
<td>293 K</td>
<td></td>
</tr>
</tbody>
</table>
Charles’ Law

Charles’ law shows a direct relationship between the volume of a gas and the temperature of a gas when the temperature is given in the **Kelvin scale**. The Charles’ law equation is below.

Converting from degrees Celsius to Kelvin is easy — you *add* 273 to the Celsius temperature. To convert from Kelvins to degrees Celsius, you *subtract* 273 from the Kelvin temperature.

**EXAMPLE**

A truck tire holds 25.0 liters of air at 25°C. If the temperature drops to 0°C, and the pressure remains constant, what will be the new volume of the tire?

**Looking for**
The new volume of the tire \((V_2)\)

**Given**
\(V_1 = 25.0\) liters; \(T_1 = 25°C; T_2 = 0°C\)

**Relationships**
Use Charles’ Law to solve for \(V_2\). Multiply each side by \(T_2\) to isolate \(V_2\) on one side of the equation.

\[
V_2 = \frac{V_1 T_2}{T_1}
\]

Convert temperature values in Celsius degrees to Kelvin: \(T_{\text{Kelvin}} = T_{\text{Celsius}} + 273\)

**Solution**
\(T_1 = 25°C + 273 = 298\)
\(T_2 = 0°C + 273 = 273\)

\[
V_2 = \frac{25.0 \text{ L} \times 273}{298} = 23.0 \text{ L}
\]

The new volume inside the tire is 23.0 liters.

**PRACTICE**

1. If a truck tire holds 25.0 liters of air at 25.0°C, what will be the volume of air in the tire if the temperature increases to 30.0°C?

2. A balloon holds 20.0 liters of helium at 10.0°C. If the temperature increases to 50.0°C, and the pressure does not change, what will be the new volume of the balloon?

3. Use Charles’ Law to fill in the following table with the correct values. Pay attention to the temperature units.

<table>
<thead>
<tr>
<th></th>
<th>(V_1)</th>
<th>(T_1)</th>
<th>(V_2)</th>
<th>(T_2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>a.</td>
<td></td>
<td>840 K</td>
<td>1,070 mL</td>
<td>147 K</td>
</tr>
<tr>
<td>b.</td>
<td>3250 mL</td>
<td>475°C</td>
<td></td>
<td>50°C</td>
</tr>
<tr>
<td>c.</td>
<td>10 L</td>
<td></td>
<td>15 L</td>
<td>50°C</td>
</tr>
</tbody>
</table>
The Structure of the Atom

Atoms are made of three tiny subatomic particles: protons, neutrons, and electrons. The protons and neutrons are grouped together in the nucleus, which is at the center of the atom. The chart below compares electrons, protons, and neutrons in terms of charge and mass.

<table>
<thead>
<tr>
<th>Occurrence</th>
<th>Charge</th>
<th>Mass (g)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Electron</td>
<td>-1</td>
<td>$9.109 \times 10^{-28}$</td>
</tr>
<tr>
<td>Proton</td>
<td>+1</td>
<td>$1.673 \times 10^{-24}$</td>
</tr>
<tr>
<td>Neutron</td>
<td>0</td>
<td>$1.675 \times 10^{-24}$</td>
</tr>
</tbody>
</table>

The **atomic number** of an element is the number of protons in the nucleus of every atom of that element.

**Isotopes** are atoms of the same element that have different numbers of neutrons. The number of protons in isotopes of an element is the same.

The **mass number** of an isotope tells you the number of protons plus the number of neutrons.

**Mass number = number of protons + number of neutrons**

The **atomic mass** of an element is based on the mass numbers of the elements isotopes. For example, a standard table of elements lists an atomic mass of 6.94 for the element lithium. That does NOT mean there are 3 protons and 3.94 neutrons in a lithium atom! On average, 94 percent of lithium atoms are lithium-7 and 6 percent are lithium-6 (Figure 9.10). The average atomic mass of lithium is 6.94 because of the weighted average of the mixture of isotope.

**EXAMPLES**

Carbon has three isotopes: carbon-12, carbon-13, and carbon-14. The atomic number of carbon is 6. The atomic mass of carbon, 12.0111 amu.

<table>
<thead>
<tr>
<th>Question</th>
<th>Answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. How many protons are in the nucleus of a carbon atom?</td>
<td><strong>Answer:</strong> 6 protons; the atomic number indicates how many protons are in the nucleus of an atom.</td>
</tr>
<tr>
<td>b. How many neutrons are in the nucleus of a carbon-13 atom?</td>
<td><strong>Answer:</strong> 7 protons&lt;br&gt;the mass number - the atomic number = the number of neutrons&lt;br&gt;13 − 6 = 7</td>
</tr>
<tr>
<td>c. Which of the carbon isotopes is most abundant in nature?</td>
<td><strong>Answer:</strong> At atomic mass (12.0111 amu) is based on the mass number and abundance of each of the carbon isotopes. Because the mass number is close to the whole number “12,” we can assume that the most abundant isotope of carbon is carbon-12 (whose mass number is 12).</td>
</tr>
</tbody>
</table>
Use a periodic table of the elements to answer these questions.

1. The following graphics represent the nuclei of atoms. Using a periodic table of elements, fill in the table.

<table>
<thead>
<tr>
<th>What the nucleus looks like</th>
<th>What is this element?</th>
<th>How many electrons does the neutral atom have?</th>
<th>What is the mass number?</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1" alt="Atom 1" /></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><img src="image2" alt="Atom 2" /></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><img src="image3" alt="Atom 3" /></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><img src="image4" alt="Atom 4" /></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2. Look at a periodic table. The atomic mass of hydrogen is 1.00794. Why is this number not rounded off to 1?

3. How many protons and neutrons are in the nucleus of each isotope?
   a. hydrogen-2 (atomic number = 1)
   b. scandium-45 (atomic number = 21)
   c. aluminum-27 (atomic number = 13)
   d. uranium-235 (atomic number = 92)
   e. carbon-12 (atomic number = 6)

4. Although electrons have mass, they are not considered in determining the mass number or atomic mass of an atom. Why?

5. A hydrogen atom has one proton, two neutrons, and no electrons. Is this atom neutrally charged? Explain your answer.

6. An atom of sodium-23 (atomic number = 11) has a positive charge of +1. Given this information, how many electrons does it have? How many protons and neutrons does this atom have?
You have learned that atoms are composed of protons, neutrons, and electrons. The electrons occupy energy levels that surround the nucleus in the form of an “electron cloud.” The electrons that are involved in forming chemical bonds are called **valence electrons**. Atoms can have up to eight valence electrons. These electrons exist in the outermost region of the electron cloud often called the “valence shell.”

The most stable atoms have eight valence electrons. When an atom has eight valence electrons, it is said to have a complete octet. Atoms will gain or lose electrons in order to complete their octet. In the process of gaining or losing electrons, atoms will form chemical bonds with other atoms. The method we use to visually represent an atom’s valence state is called a **dot diagram**, and you will practice drawing these in the following exercise.

**What is a dot diagram?**

Dot diagrams are composed of two parts—the chemical symbol for the element and dots surrounding the chemical symbol. Each dot represents one valence electron.

- If an element, such as oxygen (O), has six valence electrons, then six dots will surround the chemical symbol as shown to the right.

- Boron (B) has three valence electrons, so three dots surround the chemical symbol for boron as shown to the right.

There can be up to eight dots around a symbol, depending on the number of valence electrons the atom has. The first four dots are single, and then as more dots are added, they fill in as pairs.

Using a periodic table, complete the following chart. With this information, draw a dot diagram for each element in the chart. Remember, only the valence electrons are represented in the diagram, not the total number of electrons.

<table>
<thead>
<tr>
<th>Element</th>
<th>Chemical symbol</th>
<th>Total number of electrons</th>
<th>Number of valence electrons</th>
<th>Dot diagram</th>
</tr>
</thead>
<tbody>
<tr>
<td>Potassium</td>
<td>K</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Nitrogen</td>
<td>N</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Carbon</td>
<td>C</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Beryllium</td>
<td>Be</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Neon</td>
<td>Ne</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sulfur</td>
<td>S</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Using dot diagrams to represent chemical reactivity

Once you have a dot diagram for an element, you can predict how an atom will achieve a full valence shell. For instance, it is easy to see that chlorine has one empty space in its valence shell. It is likely that chlorine will try to gain one electron to fill this empty space rather than lose the remaining seven. However, potassium has a single dot or electron in its dot diagram. This diagram shows how much easier it is to lose this lone electron than to find seven to fill the seven empty spaces. When the potassium loses its electron, it becomes positively charged. When chlorine gains the electron, it becomes negatively charged. Opposite charges attract, and this attraction draws the atoms together to form what is termed an ionic bond, a bond between two charged atoms or ions.

Because chlorine needs one electron, and potassium needs to lose one electron, these two elements can achieve a complete set of eight valence electrons by forming a chemical bond. We can use dot diagrams to represent the chemical bond between chlorine and potassium as shown above.

For magnesium and chlorine, however, the situation is a bit different. By examining the electron or Lewis dot diagrams for these atoms, we see why magnesium requires two atoms of chlorine to produce the compound, magnesium chloride, when these two elements chemically combine.

Magnesium can easily donate one of its valence electrons to the chlorine to fill chlorine’s valence shell, but this still leaves magnesium unstable; it still has one lone electron in its valence shell. However, if it donates that electron to another chlorine atom, the second chlorine atom has a full shell, and now so does the magnesium.

The chemical formula for potassium chloride is KCl. This means that one unit of the compound is made of one potassium atom and one chlorine atom.

The formula for magnesium chloride is MgCl₂. This means that a one unit of the compound is made of one magnesium atom and two chlorine atoms.

Now try using dot diagrams to predict chemical formulas. Fill in the table below:

<table>
<thead>
<tr>
<th>Elements</th>
<th>Dot diagram for each element</th>
<th>Dot diagram for compound formed</th>
<th>Chemical formula</th>
</tr>
</thead>
<tbody>
<tr>
<td>Na and F</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Br and Br</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mg and O</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Power in Flowing Energy

**Power** is the rate of doing work. You do work if you lift a heavy box up a flight of stairs. You do the same amount of work whether you lift the box slowly or quickly. But your power is greater if you do the work in a smaller amount of time.

Power can also be used to describe the rate at which energy is converted from one form into another. A light bulb converts electrical energy into heat (thermal energy) and light (radiant energy). The power of a light bulb is the rate at which the electrical energy is converted into these other forms.

To calculate the power of a person, machine, or other device, you must know the work done or energy converted and the time. Work can be calculated using the following formula:

\[
W = F \times d
\]

Both work and energy are measured in joules. A joule is actually another name for a newton·meter. If you push an object along the floor with a force of 1 newton for a distance of 1 meter, you have done 1 joule of work. A motor could be used to do this same task by converting 1 joule of electrical energy into mechanical energy.

Power is calculated by dividing the work or energy by the time. Power is measured in watts. One watt is equal to one joule of work or energy per second. In one second, a 60-watt light bulb converts 60 joules of electrical energy into heat and light. Power can also be measured in horsepower. One horsepower is equal to 746 watts.

\[
P = \frac{W}{t}
\]

A cat who weighs 40 newtons climbs a tree that is 15 meters tall in 10 seconds. Calculate the work done by the cat and the cat’s power.

<table>
<thead>
<tr>
<th><strong>Looking for</strong></th>
<th>The work and power of the cat.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Given</strong></td>
<td>The force is 40 N.</td>
</tr>
<tr>
<td></td>
<td>The distance is 15 m.</td>
</tr>
<tr>
<td></td>
<td>The time is 10 seconds.</td>
</tr>
<tr>
<td><strong>Relationships</strong></td>
<td>Work = Force × distance</td>
</tr>
<tr>
<td></td>
<td>Power = Work/time</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th><strong>Solution</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>Work = 40 N • 15 m = 600 J</td>
</tr>
<tr>
<td>Power = \frac{600 \text{ J}}{10 \text{ sec}} = 60 \text{ W}</td>
</tr>
<tr>
<td>The work done by the cat is 600 joules.</td>
</tr>
<tr>
<td>The power of the cat is 60 watts.</td>
</tr>
<tr>
<td>In units of horsepower, the cat’s power is (60 watts)(1 hp / 746 watts) = 0.12 horsepower.</td>
</tr>
</tbody>
</table>
1. Complete the table below:

<table>
<thead>
<tr>
<th>Force (N)</th>
<th>Distance (m)</th>
<th>Time (sec)</th>
<th>Work (J)</th>
<th>Power (W)</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>2</td>
<td>5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>100</td>
<td>2</td>
<td>10</td>
<td></td>
<td></td>
</tr>
<tr>
<td>100</td>
<td>4</td>
<td>10</td>
<td></td>
<td></td>
</tr>
<tr>
<td>100</td>
<td>20</td>
<td>25</td>
<td>500</td>
<td></td>
</tr>
<tr>
<td>100</td>
<td>20</td>
<td>20</td>
<td>1000</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>30</td>
<td>10</td>
<td></td>
<td>60</td>
</tr>
<tr>
<td>3</td>
<td>20</td>
<td></td>
<td>75</td>
<td>5</td>
</tr>
</tbody>
</table>

2. Oliver weighs 600. N. He climbs a flight of stairs that is 3.0 meters tall in 4.0 seconds.
   a. How much work did he do?
   b. What was Oliver’s power in watts?

3. An elevator weighing 6,000 N moves up a distance of 10.0 meters in 30.0 seconds.
   a. How much work did the elevator’s motor do?
   b. What was the power of the elevator’s motor in watt and in horsepower?

4. After a large snowstorm, you shovel 2,500 kilograms of snow off of your sidewalk in half an hour. You lift the shovel to an average height of 1.5 meters while you are piling the snow in your yard.
   a. How much work did you do? Hint: The force is the weight of the snow.
   b. What was your power in watts? Hint: You must always convert time to seconds when calculating power.

5. A television converts 12,000 joules of electrical energy into light and sound every minute. What is the power of the television?

6. The power of a typical adult’s body over the course of a day is 100 watts. This means that 100 joules of energy from food are needed each second.
   a. An average apple contains 500,000 joules of energy. For how many seconds would an apple power a person?
   b. How many joules are needed each day?
   c. How many apples would a person need to eat to get enough energy for one day?

7. A mass of 1,000 kilograms of water drops 10.0 meters down a waterfall every second.
   a. How much potential energy is converted into kinetic energy every second?
   b. What is the power of the waterfall in watts and in horsepower

8. An alkaline AA battery stores approximately 12,000 J of energy. A small flashlight uses two AA batteries and will produce light for 2 hours. What is the power of the flashlight bulb? Assume all of the energy in the batteries is used.
Efficiency and Energy

**READ**

**Efficiency** describes how well energy is converted from one form into another. A process is 100% efficient if no energy is “lost” due to friction, to create sound, or for other reasons. In reality, no process is 100% efficient.

Efficiency is calculated by dividing the output energy by the input energy. If you multiply the result by 100, you will get efficiency as a percentage. For example, if the answer you get is 0.50, you can multiply by 100 and write your answer as 50%.

**EXAMPLE**

You drop a 2-kilogram box from a height of 3 meters. Its speed is 7 m/sec when it hits the ground. How efficiently did the potential energy turn into kinetic energy?

<table>
<thead>
<tr>
<th>Looking for</th>
<th>Solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>You are asked to find the efficiency.</td>
<td>$E_p = (2 \text{ kg})(9.8 \text{ m/sec}^2)(3 \text{ m}) = 58.8 \text{ J}$ $E_K = (1/2)(2 \text{ kg})(7 \text{ m/sec})^2 = 49 \text{ J}$ The input energy is the potential energy, and the output energy is the kinetic energy. Efficiency = $(49 \text{ J})/(58.8 \text{ J}) = 0.83 \text{ or 83%}$ The efficiency is 0.83 or 83% $(0.83 \times 100)$.</td>
</tr>
</tbody>
</table>

**PRACTICE**

1. Engineers who design battery-operated devices such as cell phones and MP3 players try to make them as efficient as possible. An engineer tests a cell phone and finds that the batteries supply 10,000 J of energy to make 5500 J of output energy in the form of sound and light for the screen. How efficient is the phone?

2. What’s the efficiency of a car that uses 400,000 J of energy from gasoline to make 48,000 J of kinetic energy?

3. A 1000 kilogram roller coaster goes down a hill that is 90 meters tall. Its speed at the bottom is 40 m/sec.
   a. What is the efficiency of the roller coaster? Assume it starts from rest at the top of the hill.
   b. What do you think happens to the “lost” energy?
   c. Use the concepts of energy and efficiency to explain why the first hill on a roller coaster is the tallest.

4. You see an advertisement for a new free fall ride at an amusement park. The ad says the ride is 50 meters tall and reaches a speed of 28 m/sec at the bottom. How efficient is the ride? Hint: You can use any mass you wish because it cancels out.

5. Imagine that you are working as a roller coaster designer. You want to build a record-breaking coaster that goes 70 m/sec at the bottom of the first hill. You estimate that the efficiency of the tracks and cars you are using is 90%. How high must the first hill be?
Balancing Chemical Equations

Chemical symbols provide us with a shorthand method of writing the name of an element. Chemical formulas do the same for compounds. But what about chemical reactions? To write out, in words, the process of a chemical change would be long and tedious. Is there a shorthand method of writing a chemical reaction so that the all the information is presented correctly and is understood by all scientists? Yes! This is the function of chemical equations. You will practice writing and balancing chemical equations in this skill sheet.

What are chemical equations?

Chemical equations show what is happening in a chemical reaction. They provide you with the identities of the reactants (substances entering the reaction) and the products (substances formed by the reaction). They also tell you how much of each substance is involved in the reaction. Chemical equations use symbols for elements and formulas for compounds. The reactants are written to the left of the arrow. Products go on the right side of the arrow.

\[ \text{H}_2 + \text{O}_2 \rightarrow \text{H}_2\text{O} \]

The arrow should be read as “yields” or “produces.” This equation, therefore, says that hydrogen gas (H\(_2\)) plus oxygen gas (O\(_2\)) yields or produces the compound water (H\(_2\)O).

**PRACTICE 1**

Write chemical equations for the following reactions:

<table>
<thead>
<tr>
<th>Reactants</th>
<th>Products</th>
<th>Chemical Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hydrochloric acid HCl and Sodium hydroxide NaOH</td>
<td>Water H(_2)O and Sodium chloride NaCl</td>
<td></td>
</tr>
<tr>
<td>Calcium carbonate CaCO(_3) and Potassium iodide KI</td>
<td>Potassium carbonate K(_2)CO(_3) and Calcium iodide CaI(_2)</td>
<td></td>
</tr>
<tr>
<td>Aluminum fluoride AlF(_3) and Magnesium nitrate Mg(NO(_3))(_2)</td>
<td>Aluminum nitrate Al(NO(_3))(_3) and Magnesium fluoride MgF(_2)</td>
<td></td>
</tr>
</tbody>
</table>
Conservation of atoms

Take another look at the chemical equation for making water:

\[ 2H_2 + O_2 \rightarrow 2H_2O \]

Did you notice that something has been added?

The large number in front of \( H_2 \) tells how many molecules of \( H_2 \) are required for the reaction to proceed. The large number in front of \( H_2O \) tells how many molecules of water are formed by the reaction. These numbers are called *coefficients*. Using coefficients, we can balance chemical equations so that the equation demonstrates conservation of atoms. The law of conservation of atoms says that no atoms are lost or gained in a chemical reaction. The same types and numbers of atoms must be found in the reactants and the products of a chemical reaction.

Coefficients are placed before the chemical symbol for single elements and before the chemical formula of compounds to show how many atoms or molecules of each substance are participating in the chemical reaction. When counting atoms to balance an equation, remember that the coefficient applies to all atoms within the chemical formula for a compound. For example, \( 5CH_4 \) means that 5 atoms of carbon and 20 atoms \( (5 \times 4) \) of hydrogen are contributed to the chemical reaction by the compound methane.

**Balancing chemical equations**

To write a chemical equation correctly, first write the equation using the correct chemical symbols or formulas for the reactants and products.

If a reaction is to occur between sodium chloride and iodine to form sodium iodide and chlorine gas, we would write:

\[ NaCl + I_2 \rightarrow NaI + Cl_2 \]

- Next, count the number of atoms of each element present on the reactant and product side of the chemical equation:

<table>
<thead>
<tr>
<th>Reactant Side of Equation</th>
<th>Element</th>
<th>Product Side of Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Na</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>Cl</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>I</td>
<td>1</td>
</tr>
</tbody>
</table>

- For the chemical equation to be balanced, the numbers of atoms of each element must be the same on either side of the reaction. This is clearly not the case with the equation above. We need coefficients to balance the equation.
First, choose one element to balance. Let’s start by balancing chlorine. Since there are two atoms of chlorine on the product side and only one on the reactant side, we need to place a “2” in front of the substance containing the chlorine, the NaCl.

\[ 2\text{NaCl} + I_2 \rightarrow \text{NaI} + \text{Cl}_2 \]

This now gives us two atoms of chlorine on both the reactant and product sides of the equation. However, it also gives us two atoms of sodium on the reactant side! This is fine—often balancing one element will temporarily unbalance another. By the end of the process, however, all elements will be balanced.

We now have the choice of balancing either the iodine or the sodium. Let's balance the iodine. (It doesn’t matter which element we choose.)

There are two atoms of iodine on the reactant side of the equation and only one on the product side. Placing a coefficient of “2” in front of the substance containing iodine on the product side:

\[ 2\text{NaCl} + I_2 \rightarrow 2\text{NaI} + \text{Cl}_2 \]

There are now two atoms of iodine on either side of the equation, and at the same time we balanced the number of sodium atoms!

In this chemical reaction, two molecules of sodium chloride react with one molecule of iodine to produce two molecules of sodium iodide and one molecule of chlorine. Our equation is balanced!

**Practice 2**

Balance the equations on the next page using the appropriate coefficients. Remember that balancing one element may temporarily unbalance another. You will have to correct the imbalance in the final equation. Check your work by counting the total number of atoms of each element—the numbers should be equal on the reactant and product sides of the equation. Remember, the equations **cannot** be balanced by changing subscript numbers!

1. \( \text{Al} + \text{O}_2 \rightarrow \text{Al}_2\text{O}_3 \)
2. \( \text{CO} + \text{H}_2 \rightarrow \text{H}_2\text{O} + \text{CH}_4 \)
3. \( \text{HgO} \rightarrow \text{Hg} + \text{O}_2 \)
4. \( \text{CaCO}_3 \rightarrow \text{CaO} + \text{CO}_2 \)
5. \( \text{C} + \text{Fe}_2\text{O}_3 \rightarrow \text{Fe} + \text{CO}_2 \)
6. \( \text{N}_2 + \text{H}_2 \rightarrow \text{NH}_3 \)
7. \( \text{K} + \text{H}_2\text{O} \rightarrow \text{KOH} + \text{H}_2 \)
8. \( \text{P} + \text{O}_2 \rightarrow \text{P}_2\text{O}_5 \)
9. \( \text{Ba(OH)}_2 + \text{H}_2\text{SO}_4 \rightarrow \text{H}_2\text{O} + \text{BaSO}_4 \)
10. \( \text{CaF}_2 + \text{H}_2\text{SO}_4 \rightarrow \text{CaSO}_4 + \text{HF} \)
11. \( \text{KClO}_3 \rightarrow \text{KClO}_4 + \text{KCl} \)
Radioactivity

There are three main types of radiation that involve the decay of the nucleus of an atom:

- **alpha radiation** \((\alpha)\): release of a helium-4 nucleus (two protons and two neutrons). We can represent helium-4 using isotope notation: \( ^4_2\text{He} \). The top number, 4, represents the mass number, and the bottom number represents the atomic number for helium, 2.

- **beta radiation** \((\beta)\): release of an electron.

- **gamma radiation** \((\gamma)\): release of an electromagnetic wave.

Half-life

The time it takes for half of the atoms in a sample to decay is called the half-life. Four kilograms of a certain substance undergo radioactive decay. Let’s calculate the amount of substance left over after 1, 2, and 3 half-lives.

- After one half-life, the substance will be reduced by half, to 2 kilograms.
- After two half-lives, the substance will be reduced by another half, to 1 kilogram.
- After three half-lives, the substance will be reduced by another half, to 0.5 kilogram.

So, if we start with a sample of mass \( m \) that decays, after a few half-lives, the mass of the sample will be:

<table>
<thead>
<tr>
<th>Number of half-lives</th>
<th>Mass left</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( \frac{1}{2}m = \frac{1}{2} \frac{m}{2} )</td>
</tr>
<tr>
<td>2</td>
<td>( \frac{1}{2} \frac{1}{2}m = \frac{1}{4}m )</td>
</tr>
<tr>
<td>3</td>
<td>( \frac{1}{2} \frac{1}{2} \frac{1}{2}m = \frac{1}{8}m )</td>
</tr>
<tr>
<td>4</td>
<td>( \frac{1}{2} \frac{1}{2} \frac{1}{2} \frac{1}{2}m = \frac{1}{16}m )</td>
</tr>
</tbody>
</table>
1. The decay series for uranium-238 and plutonium-240 are listed below. Above each arrow, write “a” for alpha decay or “b” for beta decay to indicate which type of decay took place at each step.

a. \( ^{238}\text{U} \rightarrow ^{234}\text{Th} \rightarrow ^{234}\text{Pa} \rightarrow ^{234}\text{U} \rightarrow ^{230}\text{Th} \rightarrow \)

b. \( ^{226}\text{Ra} \rightarrow ^{222}\text{Rn} \rightarrow ^{218}\text{Po} \rightarrow ^{214}\text{Pb} \rightarrow ^{214}\text{Bi} \rightarrow \)

c. \( ^{214}\text{Po} \rightarrow ^{210}\text{Pb} \rightarrow ^{210}\text{Bi} \rightarrow ^{210}\text{Po} \rightarrow ^{206}\text{Pb} \rightarrow \)

2. Fluorine-18 \(^{18}\text{F}\) has a half-life of 110 seconds. This material is used extensively in medicine. The hospital laboratory starts the day at 9 a.m. with 10 grams of \(^{18}\text{F}\).

a. How many half-lives for fluorine-18 occur in 11 minutes (660 seconds)?

b. How much of the 10-gram sample of fluorine-18 would be left after 11 minutes?

3. The isotope \(^{14}\text{C}\) has a half-life of 5,730 years. What is the fraction of \(^{14}\text{C}\) in a sample with mass, \(m\), after 28,650 years?

4. What is the half-life of this radioactive isotope that decreases to one-fourth its original amount in 18 months?

5. This diagram illustrates a formula that is used to calculate the intensity of radiation from a radioactive source. Radiation “radiates” from a source into a spherical area. Therefore, you can calculate intensity using the area of a sphere \((4\pi r^2)\). Use the formula and the diagram to help you answer the questions below.

**Intensity**

\[
I = \frac{P}{A}
\]

Area, \(A = 4\pi r^2 = 12.6 \text{ m}^2\)

Intensity, \(I = \frac{100 \text{ W}}{12.6 \text{ m}^2}\)

\(= 7.96 \text{ W/m}^2\)

a. A radiation source with a power of 1,000 watts is located at a point in space. What is the intensity of radiation at a distance of 10 meters from the source?

b. The fusion reaction \(^2\text{H} + ^3\text{H} \rightarrow ^4\text{He} + n + \text{energy}\) releases \(2.8 \times 10^{-12}\) joules of energy. How many such reactions must occur every second in order to light a 100-watt light bulb? Note that one watt equals one joule per second.
Einstein’s Formula

Everything in the universe can be categorized as either matter or energy. Einstein worked to establish a relationship between the amount of matter (mass) making up an object and the amount of energy it contains. He derived the famous equation $E = mc^2$ to relate energy and mass.

Einstein’s formula does not mean you can take an object such as a rock and easily convert its mass into energy. Einstein thought of mass as the measure of the energy contained in an object. Getting the energy from an object’s mass is another matter.

Processes during which we can observe mass becoming energy include radioactive decay and nuclear reactions. Radioactive decay occurs when the nuclei of atoms in a radioactive substance release energy in the form of radiation. The mass of the substance gradually decreases. Nuclear reactions involve the splitting of nuclei (fission) or the combining of nuclei (fusion). Mass is converted into energy during these reactions.

1. How much energy is contained in matter with a mass of 1 gram (0.001 kilogram)?
2. How much energy is contained in the mass of a 60-kilogram person?
3. Radioactive carbon-14 decays into nitrogen-14. A piece of carbon-14 that originally had a mass of 1 kilogram is later found to have a mass of 0.9999 kilogram. How much energy was released?
4. Nuclear fusion creates energy in the sun. During this process, hydrogen atoms combine to create helium. The mass of the helium created is less than the mass of the hydrogen from which it was made. The lost mass is converted to radiant energy.
   a. The sun loses $4.3 \times 10^9$ kilogram of mass every second. How much energy is released in one second?
   b. What is the power of the sun in watts?
   c. How much mass does the sun lose each year?
   d. How much energy is released in one year?
   e. Because Earth is so far from the sun, we receive only one-half of one billionth of the sun’s energy. How much energy do we get from the sun in one year?
5. The annual energy consumption for the world totals approximately $4 \times 10^{20}$ joules.
   a. How much mass would have to be converted to energy on the sun to provide this much energy?
   b. Based on your answers to question 4, do we get enough energy from the sun to be able to meet our energy needs?
Most people who work with electric circuits use a digital multimeter to measure electrical quantities. These measurements help them analyze circuits. Most multimeters measure voltage, current, and resistance. A typical multimeter is shown below:
This table summarizes how to use and interpret any digital meter in a battery circuit. Note: A component is any part of a circuit, such as a battery, a bulb, or a wire.

<table>
<thead>
<tr>
<th>Measuring Voltage</th>
<th>Measuring Current</th>
<th>Measuring Resistance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Circuit is ON</td>
<td>Circuit is ON</td>
<td>Circuit is OFF</td>
</tr>
<tr>
<td>Turn meter dial to voltage, labeled V, VDC, or ( \bar{V} )</td>
<td>Turn meter dial to current, labeled A, ADC, or ( \bar{A} )</td>
<td>Turn meter dial to resistance, labeled ( \Omega )</td>
</tr>
<tr>
<td>Connect leads to meter following meter instructions</td>
<td>Connect leads to meter following meter instructions</td>
<td>Connect leads to meter following meter instructions</td>
</tr>
<tr>
<td>Place leads at each end of component (leads are ACROSS the component)</td>
<td>Break circuit and place leads on each side of the break (meter is IN the circuit)</td>
<td>Place leads at each end of component (leads are ACROSS the component)</td>
</tr>
<tr>
<td>Measurement in VOLTS (V)</td>
<td>Measurement in AMPS (A)</td>
<td>Measurement in OHMS (( \Omega ))</td>
</tr>
<tr>
<td>Battery measurement shows relative energy provided</td>
<td>Measurement shows the value of current at the point where meter is placed</td>
<td>Measurement shows the resistance of the component</td>
</tr>
<tr>
<td>Component measurement shows relative energy used by that component</td>
<td>Current is the flow of charge through the wire</td>
<td>When the resistance is too high, the display shows OL (overload) or ( \propto ) (infinity)</td>
</tr>
</tbody>
</table>
Build a circuit containing 2 batteries and 2 bulbs in which there is only one path for the current to follow. The batteries should placed so the positive end of one is connected to the negative end of the other. This is called a *series circuit*, and it should form one large loop.

1. Draw a circuit diagram or sketch that shows all the posts in the circuit (posts are where wires and holders connect together).

2. Measure and record the voltage across each battery.

3. Measure and record the voltage across each bulb.

4. Measure and record the total voltage across both batteries.

5. Measure and record the total voltage across both bulbs.

6. How does the total voltage across the bulbs compare to the total voltage across the batteries?

7. Break the circuit at one post. Measure and record the current. Repeat until you have measured the current at every post.

8. How does the current compare at different points in the circuit?

9. Disconnect one of the bulbs from its holder. Measure and record the bulb’s resistance. Repeat with the other bulb.

10. Create a set of step-by-step instructions explaining how to use the meter to measure a bulb’s resistance, the current through it, and the voltage across it. Find someone unfamiliar with the meter. See if they can follow your instructions.
Ohm's Law

A German physicist, Georg S. Ohm, developed this mathematical relationship, which is present in most circuits. This relationship is known as Ohm's law. This relationship states that if the voltage (energy) in a circuit increases, so does the current (flow of charges). If the resistance increases, the current flow decreases.

\[
\text{Current (amps)} = \frac{\text{Voltage (volts)}}{\text{Resistance (ohms, } \Omega)}
\]

To work through this skill sheet, you will need the symbols used to depict circuits in diagrams. The symbols that are most commonly used for circuit diagrams are provided to the right.

If a circuit contains more than one battery, the total voltage is the sum of the individual voltages. A circuit containing two 6 V batteries has a total voltage of 12 V. [Note: The batteries must be connected positive to negative for the voltages to add.]

**EXAMPLE**

If a toaster produces 12 ohms of resistance in a 120-volt circuit, what is the amount of current in the circuit?

<table>
<thead>
<tr>
<th>Given</th>
<th>Solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>The resistance ((R)) is 12 ohms.</td>
<td>(I = \frac{V}{R} = \frac{120 \text{ volts}}{12 \text{ ohms}} = 10 \text{ amps})</td>
</tr>
<tr>
<td>The voltage ((V)) is 120 volts.</td>
<td>The current in the toaster circuit is 10 amps.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Looking for</th>
<th>Relationships</th>
</tr>
</thead>
<tbody>
<tr>
<td>The amount of current ((I)) in the circuit.</td>
<td>(I = \frac{V}{R})</td>
</tr>
</tbody>
</table>

If a problem asks you to calculate the voltage or resistance, you must rearrange the equation \(I=V/R\) to solve for \(V\) or \(R\). All three forms of the equation are listed below.

\[
I = \frac{V}{R} \quad V = IR \quad R = \frac{V}{I}
\]

**PRACTICE**

In this section, you will find some problems based on diagrams and others without diagrams. In all cases, show your work.

1. How much current is in a circuit that includes a 9-volt battery and a bulb with a resistance of 3 ohms?
2. How much current is in a circuit that includes a 9-volt battery and a bulb with a resistance of 12 ohms?
3. A circuit contains a 1.5 volt battery and a bulb with a resistance of 3 ohms. Calculate the current.
4. A circuit contains two 1.5 volt batteries and a bulb with a resistance of 3 ohms. Calculate the current.

5. What is the voltage of a circuit with 15 amps of current and toaster with 8 ohms of resistance?

6. A light bulb has a resistance of 4 ohms and a current of 2 A. What is the voltage across the bulb?

7. How much voltage would be necessary to generate 10 amps of current in a circuit that has 5 ohms of resistance?

8. How many ohms of resistance must be present in a circuit that has 120 volts and a current of 10 amps?

9. An alarm clock draws 0.5 A of current when connected to a 120 volt circuit. Calculate its resistance.

10. A portable CD player uses two 1.5 V batteries. If the current in the CD player is 2 A, what is its resistance?

11. You have a large flashlight that takes 4 D-cell batteries. If the current in the flashlight is 2 amps, what is the resistance of the light bulb? (Hint: A D-cell battery has 1.5 volts.)

12. Use the diagram below to answer the following problems.

   ![Diagram](image)

   a. What is the total voltage in each circuit?
   b. How much current would be measured in each circuit if the light bulb has a resistance of 6 ohms?
   c. How much current would be measured in each circuit if the light bulb has a resistance of 12 ohms?
   d. Is the bulb brighter in circuit A or circuit B? Why?

13. What happens to the current in a circuit if a 1.5-volt battery is removed and is replaced by a 9-volt battery?


15. In your own words, state the relationship between voltage and current in a circuit.

16. What could you do to a closed circuit consisting of 2 batteries, 2 light bulbs, and a switch to increase the current? Explain your answer.

17. What could you do to a closed circuit consisting of 2 batteries, 2 light bulbs, and a switch to decrease the current? Explain your answer.

18. You have four 1.5 V batteries, a 1 Ω bulb, a 2 Ω bulb, and a 3 Ω bulb. Draw a circuit you could build to create each of the following currents. There may be more than one possible answer for each.

   a. 1 ampere
   b. 2 amperes
   c. 3 amperes
   d. 6 amperes
In a series circuit, current follows only one path from the positive end of the battery toward the negative end. The total resistance of a series circuit is equal to the sum of the individual resistances. The amount of energy used by a series circuit must equal the energy supplied by the battery. In this way, electrical circuits follow the law of conservation of energy. Understanding these facts will help you solve problems that deal with series circuits.

To answer the questions in the practice section, you will have to use Ohm's law. Remember that:

\[
\text{Current (amps)} = \frac{\text{Voltage (volts)}}{\text{Resistance (ohms)}}
\]

Some questions ask you to calculate a voltage drop. We often say that each resistor (or light bulb) creates a separate voltage drop. As current flows along a series circuit, each resistor uses up some energy. As a result, the voltage gets lower after each resistor. If you know the current in the circuit and the resistance of a particular resistor, you can calculate the voltage drop using Ohm’s law.

Voltage drop across resistor (volts) = Current through resistor (amps) \times Resistance of one resistor (ohms)

**PRACTICE**

1. Use the series circuit pictured to the right to answer questions (a)-(e).
   a. What is the total voltage across the bulbs?
   b. What is the total resistance of the circuit?
   c. What is the current in the circuit?
   d. What is the voltage drop across each light bulb? (Remember that voltage drop is calculated by multiplying current in the circuit by the resistance of a particular resistor: \( V = IR \).)
   e. Draw the path of the current on the diagram.

2. Use the series circuit pictured to the right to answer questions (a)-(e).
   a. What is the total voltage across the bulbs?
   b. What is the total resistance of the circuit?
   c. What is the current in the circuit?
   d. What is the voltage drop across each light bulb?
   e. Draw the path of the current on the diagram.

3. What happens to the current in a series circuit as more light bulbs are added? Why?

4. What happens to the brightness of each bulb in a series circuit as additional bulbs are added? Why?
5. Use the series circuit pictured to the right to answer questions (a), (b), and (c).

   a. What is the total resistance of the circuit?
   b. What is the current in the circuit?
   c. What is the voltage drop across each resistor?

6. Use the series circuit pictured to the right to answer questions (a)-(e).

   a. What is the total voltage of the circuit?
   b. What is the total resistance of the circuit?
   c. What is the current in the circuit?
   d. What is the voltage drop across each light bulb?
   e. Draw the path of the current on the diagram.

7. Use the series circuit pictured right to answer questions (a), (b), and (c). Consider each resistor equal to all others.

   a. What is the resistance of each resistor?
   b. What is the voltage drop across each resistor?
   c. On the diagram, show the amount of voltage in the circuit before and after each resistor.

8. Use the series circuit pictured right to answer questions (a) - (d).

   a. What is the total resistance of the circuit?
   b. What is the current in the circuit?
   c. What is the voltage drop across each resistor?
   d. What is the sum of the voltage drops across the three resistors? What do you notice about this sum?

9. Use the diagram to the right to answer questions (a), (b), and (c).

   a. How much current would be measured in each circuit if each light bulb has a resistance of 6 ohms?
   b. How much current would be measured in each circuit if each light bulb has a resistance of 12 ohms?
   c. What happens to the amount of current in a series circuit as the number of batteries increases?
A parallel circuit has at least one point where the circuit divides, creating more than one path for current. Each path is called a branch. The current through a branch is called branch current. If current flows into a branch in a circuit, the same amount of current must flow out again. This rule is known as **Kirchoff’s current law**.

Because each branch in a parallel circuit has its own path to the battery, the voltage across each branch is equal to the battery’s voltage. If you know the resistance and voltage of a branch you can calculate the current with Ohm’s Law \( I = \frac{V}{R} \).

**Practice 1**

1. Use the parallel circuit pictured right to answer questions (a) - (d).
   - a. What is the voltage across each bulb?
   - b. What is the current in each branch?
   - c. What is the total current provided by the battery?
   - d. Use the total current and the total voltage to calculate the total resistance of the circuit.

2. Use the parallel circuit pictured right to answer questions (a) - (d).
   - a. What is the voltage across each bulb?
   - b. What is the current in each branch?
   - c. What is the total current provided by the battery?
   - d. Use the total current and the total voltage to calculate the total resistance of the circuit.

3. Use the parallel circuit pictured right to answer questions (a) - (d).
   - a. What is the voltage across each resistor?
   - b. What is the current in each branch?
   - c. What is the total current provided by the batteries?
   - d. Use the total current and the total voltage to calculate the total resistance of the circuit.

4. Use the parallel circuit pictured right to answer questions (a) - (c).
   - a. What is the voltage across each resistor?
   - b. What is the current in each branch?
   - c. What is the total current provided by the battery?
In part (d) of problems 1, 2, and 3, you calculated the total resistance of each circuit. This required you to first find the current in each branch. Then you found the total current and used Ohm’s law to calculate the total resistance. Another way to find the total resistance of two parallel resistors is to use the formula shown below.

\[ R_{total} = \frac{R_1 \times R_2}{R_1 + R_2} \]

**EXAMPLE**

Calculate the total resistance of a circuit containing two 6 ohm resistors.

<table>
<thead>
<tr>
<th>Given</th>
<th>Solution</th>
</tr>
</thead>
</table>
| The circuit contains two 6 Ω resistors in parallel. | \[ R_{total} = \frac{6 \ \Omega \times 6 \ \Omega}{6 \ \Omega + 6 \ \Omega} \]
| The total resistance of the circuit. | \[ R_{total} = 3 \ \Omega \] |

The total resistance is 3 ohms.

**PRACTICE 2**

1. Calculate the total resistance of a circuit containing each of the following combinations of resistors.
   a. Two 8 Ω resistors in parallel
   b. Two 12 Ω resistors in parallel
   c. A 4 Ω resistor and an 8 Ω resistor in parallel
   d. A 12 Ω resistor and a 3 Ω resistor in parallel

2. To find the total resistance of three resistors A, B, and C in parallel, first use the formula to find the total of resistors A and B. Then use the formula again to combine resistor C with the total of A and B. Use this method to find the total resistance of a circuit containing each of the following combinations of resistors.
   a. Three 8 Ω resistors in parallel
   b. Two 6 Ω resistors and a 2 Ω resistor in parallel
   c. A 1 Ω, a 2 Ω, and a 3 Ω resistor in parallel
Electrical Power

How do you calculate electrical power?

In this skill sheet you will review the relationship between electrical power and Ohm’s law. As you work through the problems, you will practice calculating the power used by common appliances in your home.

During everyday life we hear the word **watt** mentioned in reference to things like light bulbs and electric bills. The watt is the unit that describes the rate at which energy is used by an electrical device. Energy is never created or destroyed, so “used” means it is converted from electrical energy into another form such as light or heat. And since energy is measured in joules, power is measured in joules per second. One joule per second is equal to one watt.

We can calculate the amount of electrical power by an appliance or other electrical component by multiplying the voltage by the current.

\[
\text{Current} \times \text{Voltage} = \text{Power}, \text{ or } P = IV
\]

A kilowatt (kWh) is 1,000 watts or 1,000 joules of energy per second. On an electric bill you may have noticed the term kilowatt-hour. A kilowatt-hour means that one kilowatt of power has been used for one hour. To determine the kilowatt-hours of electricity used, multiply the number of kilowatts by the time in hours.

**EXAMPLE**

You use a 1500 watt hair heater for 3 hours. How many kilowatt-hours of electricity did you use?

<table>
<thead>
<tr>
<th>Given</th>
<th>Solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>The power of the heater is 1500 watts.</td>
<td>[ 1500 \text{ watts} \times \frac{1 \text{ kilowatt}}{1000 \text{ watts}} = 1.5 \text{ kilowatts} ]</td>
</tr>
<tr>
<td>The heater was used for 3 hours.</td>
<td>[ 1.5 \text{ kilowatts} \times 3 \text{ hours} = 4.5 \text{ kilowatt-hours} ]</td>
</tr>
</tbody>
</table>

Looking for

The number of kilowatt-hours.

Relationships

kilowatt-hours = kilowatts x hours

You used 4.5 kilowatt-hours of electricity.

**PRACTICE**

1. Your oven has a power rating of 5000 watts.
   a. How many kilowatts is this?
   b. If the oven is used for 2 hours to bake cookies, how many kilowatt-hours (kWh) are used?
   c. If your town charges $0.15/kWh, what is the cost to use the oven to bake the cookies?

2. You use a 1200-watt hair dryer for 10 minutes each day.
   a. How many minutes do you use the hair dryer in a month? (Assume there are 30 days in the month.)
   b. How many hours do you use the hair dryer in a month?
   c. What is the power of the hair dryer in kilowatts?
   d. How many kilowatt-hours of electricity does the hair dryer use in a month?
   e. If your town charges $0.15/kWh, what is the cost to use the hair dryer for a month?
3. Calculate the power rating of a home appliance (in kilowatts) that uses 8 amps of current when plugged into a 120-volt outlet.

4. Calculate the power of a motor that draws a current of 2 A when connected to a 12 volt battery.

5. Your alarm clock is connected to a 120 volt circuit and draws 0.5 A of current.
   a. Calculate the power of the alarm clock in watts.
   b. Convert the power to kilowatts.
   c. Calculate the number of kilowatt-hours of electricity used by the alarm clock if it is left on for one year.
   d. Calculate the cost of using the alarm clock for one year if your town charges $0.15/kWh.

6. Using the formula for power, calculate the amount of current through a 75-watt light bulb that is connected to a 120-volt circuit in your home.

7. The following questions refer to the diagram.
   a. What is the total voltage of the circuit?
   b. What is the current in the circuit?
   c. What is the power of the light bulb?

8. A toaster is plugged into a 120-volt household circuit. It draws 5 amps of current.
   a. What is the resistance of the toaster?
   b. What is the power of the toaster in watts?
   c. What is the power in kilowatts?

9. A clothes dryer in a home has a power of 4,500 watts and runs on a special 220-volt household circuit.
   a. What is the current through the dryer?
   b. What is the resistance of the dryer?
   c. How many kilowatt-hours of electricity are used by the dryer if it is used for 4 hours in one week?
   d. How much does it cost to run the dryer for one year if it is used for 4 hours each week at a cost of $0.15/kWh?

10. A circuit contains a 12-volt battery and two 3-ohm bulbs in series.
    a. Calculate the total resistance of the circuit.
    b. Calculate the current in the circuit.
    c. Calculate the power of each bulb.
    d. Calculate the power supplied by the battery.

11. A circuit contains a 12-volt battery and two 3-ohm bulbs in parallel.
    a. What is the voltage across each branch?
    b. Calculate the current in each branch.
    c. Calculate the power of each bulb.
    d. Calculate the total current in the circuit.
    e. Calculate the power supplied by the battery.
Coulomb’s Law

In this skill sheet, you will work with Coulomb’s law. There are many similarities and some differences between the equation of universal gravitation and the equation for Coulomb’s law. They are both inverse square law relationships, and they both have similar arrangements of variables.

When two charges $q_1$ and $q_2$ are separated by a distance $r$, there exists a force between them that is given by:

$$F = \frac{K q_1 q_2}{r^2}$$

where $F$ equals the force in newtons and $K$ is a constant equal to $9 \times 10^9$ N·m²/C². The units of $q_1$ and $q_2$ are the coulombs (C). Distance is given in meters. Here are some important points about the relationships of the variables in Coulomb’s law.

- Force is inversely proportional to the square of the distance between the charges. Therefore, if the distance increases by a factor of 2, the force decreases by a factor of 4.
- Force is proportional to the strength of each charge.
- When the two charges have the same sign (positive or negative), the force between them is repulsive because like charges repel.
- When the charges have opposite signs, the force between them is attractive because unlike charges attract.

1. What happens to the force between two charges if the distance between them is tripled?
2. What happens to the force between two charges if the distance between them is quadrupled?
3. What happens to the force between two charges if the distance between them is cut in half?
4. What happens to the force between two charges if the magnitude of one charge is doubled?
5. What happens to the force between two charges if the magnitude of both charges is doubled?
6. What happens to the force between two charges if the magnitude of both charges is doubled and the distance between them is doubled?
7. What happens to the force between two charges if the magnitude of both charges is doubled and the distance between them is cut in half?
The example below shows how to use Coulomb’s law to calculate the strength of the force between two charges.

A 0.001 coulomb charge and a 0.002 coulomb charge are 2 meters apart. Calculate the force between them.

### Given
- The charges have magnitudes of 0.003 C and 0.005 C.
- The charges are 2 meters apart.

### Looking for
- The force between the charges.

### Relationships
\[ F = \frac{k q_1 q_2}{r^2} \]

### Solution
\[ F = (9 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2) \frac{(0.001 \text{ C})(0.002 \text{ C})}{(2 \text{ m})^2} \]
\[ F = 4500 \text{ N} \]

The force is 4500 newtons.

---

**PRACTICE 2**

1. Two particles, each with a charge of 1 C, are separated by a distance of 1 meter. What is the force between the particles?

2. What is the force between a 3 C charge and a 2 C charge separated by a distance of 5 meters?

3. Calculate the force between a 0.006 C charge and a 0.001 C charge 4 meters apart.

4. Calculate the force between a 0.05 C charge and a 0.03 C charge 2 meters apart.

5. Two particles are each given a charge of $5 \times 10^{-5}$ C. What is the force between the charged particles if the distance between them is 2 meters?

6. The force between a pair of charges is 100 newtons. The distance between the charges is 0.01 meter. If one of the charges is $2 \times 10^{-10}$ C, what is the strength of the other charge?

7. Two equal charges separated by a distance of 1 meter experience a repulsive force of 1,000 newtons. What is the strength in coulombs of each charge?

8. The force between a pair of 0.001 C charges is 200 N. What is the distance between them?

9. The force between two charges is 1000 N. One has a charge of $2 \times 10^{-5}$ C, and the other has a charge of $5 \times 10^{-6}$ C. What is the distance between them?

10. The force between two charges is 2 newtons. The distance between the charges is $2 \times 10^{-4}$ m. If one of the charges is $3 \times 10^{-6}$ C, what is the strength of the other charge?
Earth’s magnetic field is very weak compared with the strength of the field on the surface of the ceramic magnets you probably have in your classroom. The gauss is a unit used to measure the strength of a magnetic field. A small ceramic permanent magnet has a field of a few hundred up to 1,000 gauss at its surface. At Earth’s surface, the magnetic field averages about 0.5 gauss. Of course, the field is much stronger nearer to the core of the planet.

1. What is the source of Earth’s magnetic field according to what you have read in chapter 16?

2. Today, Earth’s magnetic field is losing approximately 7 percent of its strength every 100 years. If the strength of Earth’s magnetic field at its surface is 0.5 gauss today, what will it be 100 years from now?

3. Describe what might happen if Earth’s magnetic field continues to lose strength.

4. The graphic to the right illustrates one piece of evidence that proves the reversal of Earth’s poles during the past millions of years. The ‘crust’ of Earth is a layer of rock that covers Earth’s surface. There are two kinds of crust—continental and oceanic. Oceanic crust is made continually (but slowly) as magma from Earth’s interior erupts at the surface. Newly formed crust is near the site of eruption and older crust is at a distance from the site. Based on what you know about magnetism, why might oceanic crust rock be a record of the reversal of Earth’s magnetic field? (HINT: What happens to materials when they are exposed to a magnetic field?)

5. The terms magnetic south pole and geographic north pole refer to locations on Earth. The magnetic south pole is the point on the surface above Earth’s magnetic south pole is if you think of Earth as a giant bar magnet. Geographic north is the point on the surface that we think of as ‘north.’ Explain these terms by answering the following questions.
   a. Are the locations of the magnetic south pole and the geographic north pole near Antarctica or the Arctic?
   b. How far is the magnetic south pole from the geographic north pole?
   c. In your own words, define the difference between the magnetic south pole and the geographic north pole.

6. A compass is a magnet and Earth is a magnet. How does the magnetism of a compass work with the magnetism of Earth so that a compass is a useful tool for navigating?
7. The directions—north, east, south, and west—are arranged on a compass so that they align with 360 degrees. This means that zero degrees (0°) and 360° both represent north. For each of the following directions by degrees, write down the direction in words. The first one is done for you.

a. 45°  *Answer:* The direction is northeast.

b. 180°

c. 270°

d. 90°

e. 135°

f. 315°

**Magnetic declination**

Earth’s geographic north pole (true north) and magnetic south pole are located near each other, but they are not at the same exact location. Because a compass needle is attracted to the magnetic south pole, it points slightly east or west of true north. The angle between the direction a compass points and the direction of the geographic north pole is called *magnetic declination*. Magnetic declination is measured in degrees and is indicated on topographical maps.

8. Let’s say you were hiking in the woods and relying on a compass to navigate. What would happen if you didn’t correct your compass for magnetic declination?

9. Are there places on Earth where magnetic declination equals 0°? Research on the Internet where on the globe you would have to be to have 0° magnetic declination.
A transformer is a device used to change voltage and current. You may have noticed the gray electrical boxes often located between two houses or buildings. These boxes protect the transformers that “step down” high voltage from power lines (13,800 volts) to standard household voltage (120 volts).

How a transformer works:

1. The primary coil is connected to outside power lines. Current in the primary coil creates a magnetic field through the secondary coil.

2. The current in the primary coil changes frequently because it is alternating current.

3. As the current changes, so does the strength and direction of the magnetic field through the secondary coil.

4. The changing magnetic field through the secondary coil induces current in the secondary coil. The secondary coil is connected to the wiring in your home.

Transformers work because the primary and secondary coils have different numbers of turns. If the secondary coil has fewer turns, the induced voltage in the secondary coil is lower than the voltage applied to the primary coil. You can use the proportion below to figure out how number of turns affects voltage:

\[
\frac{V_1}{V_2} = \frac{N_1}{N_2}
\]
A transformer steps down the power line voltage (13,800 volts) to standard household voltage (120 volts). If the primary coil has 5,750 turns, how many turns must the secondary coil have?

Solution:

\[
\frac{V_1}{V_2} = \frac{N_1}{N_2}
\]

\[
\begin{align*}
13,800 \text{ volts} & = 5,750 \text{ turns} \\
120 \text{ volts} & = N_2 \\
N_2 & = 50 \text{ turns}
\end{align*}
\]

1. In England, standard household voltage is 240 volts. If you brought your own hair dryer on a trip there, you would need a transformer to step down the voltage before you plug in the appliance. If the transformer steps down voltage from 240 to 120 volts, and the primary coil has 50 turns, how many turns does the secondary coil have?

2. You are planning a trip to Singapore. Your travel agent gives you the proper transformer to step down the voltage so you can use your electric appliances there. Curious, you open the case and find that the primary coil has 46 turns and the secondary has 24 turns. Assuming the output voltage is 120 volts, what is the standard household voltage in Singapore?

3. A businessman from Zimbabwe buys a transformer so that he can use his own electric appliances on a trip to the United States. The input coil has 60 turns while the output coil has 110 turns. Assuming the input voltage is 120 volts, what is the output voltage necessary for his appliances to work properly? (This is the standard household output voltage in Zimbabwe.)

4. A family from Finland, where standard household voltage is 220 volts, is planning a trip to Japan. The transformer they need to use their appliances in Japan has an input coil with 250 turns and an output coil with 550 turns. What is the standard household voltage in Japan?

5. An engineer in India (standard household voltage = 220 volts) is designing a transformer for use on her upcoming trip to Canada (standard household voltage = 120 volts). If her input coil has 240 turns, how many turns should her output coil have?

6. While in Canada, the engineer buys a new electric toothbrush. When she returns home she designs another transformer so she can use the toothbrush in India. This transformer also has an input coil with 240 turns. How many turns should the output coil have?
The Inverse Square Law

If you stand one meter away from a portable stereo blaring your favorite music, the intensity of the sound may hurt your ears. As you back away from the stereo, the sound’s intensity decreases. When you are two meters away, the sound intensity is one-fourth its original value. When you are ten meters away, the sound intensity is one-one hundredth its original value.

The sound intensity decreases according to the **inverse square law**. This means that the intensity decreases as the square of the distance. If you triple your distance from the stereo, the sound intensity decreases by nine times its original value.

Many fields follow an inverse square law, including sound, light from a small source (like a match or light bulb), gravity, and electricity. Magnetic fields are the exception. They decrease much faster because there are two magnetic poles.

**Examples**

**Example 1:** The light intensity one meter from a bulb is 2 W/m². What is the light intensity measured from a distance of four meters from the bulb?

**Solution:** The distance has increased to four times its original value. The light intensity will decrease by 4² or 16, times.

$$2 \times \frac{1}{16} = \frac{1}{8} \text{ or } 0.125 \text{ W/m}^2$$

**Example 2:** Mercury has a gravitational force of 3.7 N/kg. Its radius is 2,439 kilometers. How far away from the surface of Mercury would you need to move in order to experience a gravitational force of 0.925 N/kg?

**Solution:** For the gravitational force to be reduced to one-fourth its original value, the distance from Mercury’s center must be doubled. Therefore you would have to move to a spot 2,439 kilometers away from Mercury’s surface or 4,878 meters from its center.

**Practice**

1. You stand 4 meters away from a light and measure the intensity to be 1 W/m². What will be the intensity if you move to a position 8 meters away from the bulb?

2. You are standing 1 meter from a squawking parrot. If you move to a distance three meters away, the sound intensity will be what fraction of its original value?

3. Venus has a gravitational force of 8.9 N/kg. Its radius is 6,051 kilometers. How far away from the surface of Mercury would you need to move in order to experience a gravitational force of 0.556 N/kg?

4. Earth’s radius is 6,378 kilometers. If you weigh 500 newtons on Earth’s surface, what would you weigh at a distance of 19,134 kilometers from Earth?

5. Compare the intensity of light 2 meters away from a lit match to the intensity 6 meters away from the match.

6. How does the strength of a sound field 1 meter from its source compare with its strength 4 meters away?
Calculating Gravitational Field Strength

If we know the mass and the radius of a planet, star, or other object, we can calculate the strength of its gravitational field using this formula:

$$\text{Gravitational field, } g, \text{ in N/kg} = \frac{Gm}{r^2}, \text{ where } G = 6.67 \times 10^{-11} \text{ N}\cdot\text{m}^2/\text{kg}^2,$$

$m$ is the object's mass in kilograms and $r$ is its radius in meters.

**Example**

The planet Mercury has a radius of 2,439,000 meters and a mass of $3.3 \times 10^{23}$ kilograms. Calculate the strength of its gravitational field.

**Solution:**

$$g = \frac{Gm}{r^2} = \frac{6.67 \times 10^{-11}(3.3 \times 10^{23})}{(2.44 \times 10^6)^2} = 3.70 \text{ N/kg}$$

**Practice**

1. Venus has a radius of 6,051,000 meters and a mass of $4.9 \times 10^{24}$ kilograms. Calculate the strength of its gravitational field.

2. Jupiter has a radius of 71,398,000 meters and a mass of $1.9 \times 10^{27}$ kilograms. Calculate the strength of its gravitational field.

3. Saturn has a radius of 60,330,000 meters and a mass of $5.7 \times 10^{26}$ kilograms. Calculate the strength of its gravitational field.

4. Neptune has a radius of 24,750,000 meters and a mass of $1.0 \times 10^{26}$ kilograms. Calculate the strength of its gravitational field.

5. Uranus has a gravitational field strength of 8.7 N/kg. Its radius is 25,600,000 meters. What is the planet’s mass?

6. Earth has a gravitational field strength of 9.8 N/kg. Its radius is 6,378,000 meters. What is Earth’s mass?

7. The radius of the sun is 700,000,000 meters. Its mass is $1.99 \times 10^{30}$ kilograms. Calculate the strength of its gravitational field.
Calculating Electric Fields and Forces

Electric field strength can be described in two ways. Sometimes scientists describe electric field strength in newtons per coulomb of charge (N/C). However, in many situations the unit volts per meter (V/m) is used. The two units are equivalent. Later, you will be given an opportunity to figure out why this is true.

The force on a charge in an electric field is equal to the charge in coulombs multiplied by the electric field strength. This equation is written as:

\[
\text{Electric force in newtons (F) = charge in coulombs (q) \times electric field strength (E)}
\]

or

\[
F = qE
\]

**EXAMPLES**

**Example 1:** The electric field strength in a region is 2,200 N/C. What is the force on an object with a charge of 0.0040 C?

**Solution:**

\[
F = qE
\]

\[
F = (0.0040 \text{ C})(2,200 \text{ N/C}) = 8.8 \text{ newtons}
\]

**Example 2:** If an object with a charge of 0.080 C experiences an electric force of 7.0 N, what is the electric field strength?

**Solution:**

\[
E = \frac{F}{q}
\]

\[
E = \frac{7.0 \text{ N}}{0.080 \text{ C}} = 88 \text{ N/C}
\]

**PRACTICE**

1. What is the force of an electric field of strength 4.0 N/C on a charge of 0.5 C?
2. An electric field has a strength of 2.0 N/C. What force does it exert on a charge of 0.004 C?
3. A charge of 0.01 C is in a 120 N/C electric field. What is the force on the charge?
4. If an object with a charge of 0.08 C experiences an electric force of 5.0 N, what is the electric field strength?
5. An object with a charge of \(4.0 \times 10^{-9}\) C experiences a force of \(20 \times 10^{-9}\) N when placed in a certain point in an electric field. What is the electric field strength at that point in N/C?
6. A charge of 0.2 C experiences an electric force of 5 N. What is the strength of the electric field in N/C?
7. **Challenge!** Given that 1 joule = 1 newton \times 1 meter and 1 volt = 1 joule per coulomb, show that the units newtons/coulomb and volts/meter are equivalent.
Period and Frequency

**The period** of a pendulum is the time it takes to move through one cycle. As the ball on the string is pulled to one side and then let go, the ball moves to the side opposite the starting place and then returns to the start. This entire motion equals one cycle.

**Frequency** is a term that refers to how many cycles can occur in one second. For example, the frequency of the sound wave that corresponds to the musical note “A” is 440 cycles per second or 440 hertz. The unit hertz (Hz) is defined as the number of cycles per second.

The terms period and frequency are related by the following equation:

\[
\text{Period (seconds)} = \frac{1}{\text{Frequency (hertz)}}
\]

**Practice**

1. A string vibrates at a frequency of 20 Hz. What is its period?
2. A speaker vibrates at a frequency of 200 Hz. What is its period?
3. A swing has a period of 10 seconds. What is its frequency?
4. A pendulum has a period of 0.3 second. What is its frequency?
5. You want to describe the harmonic motion of a swing. You find out that it takes 2 seconds for the swing to complete one cycle. What is the swing’s period and frequency?
6. An oscillator makes four vibrations in one second. What is its period and frequency?
7. A pendulum takes 0.5 second to complete one cycle. What is the pendulum’s period and frequency?
8. A pendulum takes 10 seconds to swing through 2 complete cycles.
   a. How long does it take to complete one cycle?
   b. What is its period?
   c. What is its frequency?
9. An oscillator makes 360 vibrations in 3 minutes.
   a. How many vibrations does it make in one minute?
   b. How many vibrations does it make in one second?
   c. What is its period in seconds?
   d. What is its frequency in hertz?
Harmonic Motion Graphs

A graph can be used to show the amplitude and period of an object in harmonic motion. An example of a graph of a pendulum’s motion is shown below.

The distance to which the pendulum moves away from this center point is called the **amplitude**. The amplitude of a pendulum can be measured in units of length (centimeters or meters) or in degrees. On a graph, the amplitude is the distance from the x-axis to the highest point of the graph. The pendulum shown above moves 20 cm to each side of its center position, so its amplitude is 20 cm.

The **period** is the time for the pendulum to make one complete cycle. It is the time from one peak to the next on the graph. On the graph above, one peak occurs at 1.5 seconds, and the next peak occurs at 3.0 seconds. The period is $3.0 - 1.5 = 1.5$ seconds.

**PRACTICE**

1. Use the graphs to answer the following questions

   ![Angle vs Time](image1) ![Position vs Time](image2)

   a. What is the amplitude of each vibration?
   b. What is the period of each vibration?
2. Use the grids below to draw the following harmonic motion graphs. Be sure to label the $y$-axis to indicate the measurement scale.
   
   a. A pendulum with an amplitude of 2 cm and a period of 1 second.

   ![Graph 1](image1)

   b. A pendulum with an amplitude of 5 degrees and a period of 4 seconds.

   ![Graph 2](image2)
Waves

A wave is a traveling oscillator that carries energy from one place to another. A high point of a wave is called a crest. A low point is called a trough. The amplitude of a wave is half the distance from a crest to a trough. The distance from one crest to the next is called the wavelength. Wavelength can also be measured from trough to trough or from any point on the wave to the next place where that point occurs.

\[ \text{The speed of a wave} \]
\[ \text{Speed (m/sec)} \rightarrow v = f \cdot \lambda \]

\[ \text{Frequency (hertz)} \]
\[ \text{Wavelength (meters)} \]

EXAMPLE

The frequency of a wave is 40 Hz and its speed is 100 meters per second. What is the wavelength of this wave?

<table>
<thead>
<tr>
<th>Given</th>
<th>Solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency = 40 Hz; Speed = 100 m/sec</td>
<td>[ \frac{100 \text{ m/sec}}{40 \text{ Hz}} = \frac{100 \text{ m/sec}}{40 \text{ cycles/sec}} = \text{Wavelength} ]</td>
</tr>
<tr>
<td>Looking for</td>
<td>The wavelength</td>
</tr>
<tr>
<td>The wavelength</td>
<td>2.5 meters = Wavelength</td>
</tr>
<tr>
<td>Relationships</td>
<td>The wavelength of this wave is 2.5 meters.</td>
</tr>
<tr>
<td>Speed = Frequency × Wavelength, therefore</td>
<td>Speed ÷ Frequency = Wavelength</td>
</tr>
</tbody>
</table>

PRACTICE

1. On the graphic at right label the following parts of a wave: one wavelength, half of a wavelength, the amplitude, a crest, and a trough.
   a. How many wavelengths are represented in the wave above?
   b. What is the amplitude of the wave shown above?
2. Use the grids below to draw the following waves. Be sure to label the $y$-axis to indicate the measurement scale.

   a. A wave with an amplitude of 1 cm and a wavelength of 2 cm

   b. A wave with an amplitude of 1.5 cm and a wavelength of 3 cm

3. A water wave has a frequency of 2 hertz and a wavelength of 5 meters. Calculate its speed.

4. A wave has a speed of 50 m/sec and a frequency of 10 Hz. Calculate its wavelength.

5. A wave has a speed of 30 m/sec and a wavelength of 3 meters. Calculate its frequency.

6. A wave has a period of 2 seconds and a wavelength of 4 meters. Calculate its frequency and speed. 
   *Note: Recall that the frequency of a wave equals $1/\text{period}$ and the period of a wave equals $1/\text{frequency}$.*

7. A sound wave travels at 330 m/sec and has a wavelength of 2 meters. Calculate its frequency and period.

8. The frequency of wave A is 250 hertz and the wavelength is 30 centimeters. The frequency of wave B is 260 hertz and the wavelength is 25 centimeters. Which is the faster wave?

9. The period of a wave is equal to the time it takes for one wavelength to pass by a fixed point. You stand on a pier watching water waves and see 10 wavelengths pass by in a time of 40 seconds.
   
   a. What is the period of the water waves?

   b. What is the frequency of the water waves?

   c. If the wavelength is 3 meters, what is the wave speed?
Standing Waves

A wave that is confined in a space is called a standing wave. Standing waves on the vibrating strings of a guitar produce the sounds you hear. Standing waves are also present inside the chamber of a wind instrument.

A string that contains a standing wave is an oscillator. Like any oscillator, it has natural frequencies. The lowest natural frequency is called the fundamental. Other natural frequencies are called harmonics. The first five harmonics of a standing wave on a string are shown to the right.

There are two main parts of a standing wave. The nodes are the points where the string does not move at all. The antinodes are the places where the string moves with the greatest amplitude.

The wavelength of a standing wave can be found by measuring the length of two of the “bumps” on the string. The first harmonic only contains one bump, so the wavelength is twice the length of the individual bump.

**PRACTICE**

1. Use the graphic below to answer these questions.
   a. Which harmonic is shown in each of the strings below?
   b. Label the nodes and antinodes on each of the standing waves shown below.
   c. How many wavelengths does each standing wave contain?
   d. Determine the wavelength of each standing wave.
Two students want to use a 12-meter long rope to create standing waves. They first measure the speed at which a single wave pulse moves from one end of the rope to another and find that it is 36 m/sec. This information can be used to determine the frequency at which they must vibrate the rope to create each harmonic. Follow the steps below to calculate these frequencies.

a. Draw the standing wave patterns for the first six harmonics.

b. Determine the wavelength for each harmonic on the 12 meter rope. Record the values in the table below.

c. Use the equation for wave speed \((v = f\lambda)\) to calculate each frequency.

<table>
<thead>
<tr>
<th>Harmonic</th>
<th>Speed (m/sec)</th>
<th>Wavelength (m)</th>
<th>Frequency (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>36</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>36</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>36</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>36</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>36</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>36</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

d. What happens to the frequency as the wavelength increases?

e. Suppose the students cut the rope in half. The speed of the wave on the rope only depends on the material from which the rope is made and its tension, so it will not change. Determine the wavelength and frequency for each harmonic on the 6 meter rope.

<table>
<thead>
<tr>
<th>Harmonic</th>
<th>Speed (m/sec)</th>
<th>Wavelength (m)</th>
<th>Frequency (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>36</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>36</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>36</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>36</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>36</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>36</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

f. What effect did using a shorter rope have on the wavelength and frequency?
Wave Interference

Interference occurs when two or more waves are at the same location at the same time. For example, the wind may create tiny ripples on top of larger waves in the ocean. The superposition principle states that the total vibration at any point is the sum of the vibrations produced by the individual waves.

Constructive interference is when waves combine to make a larger wave. Destructive interference is when waves combine to make a wave that is smaller than either of the individual waves. Noise cancelling headphones work by producing a sound wave that perfectly cancels the sounds in the room.

This worksheet will allow you to find the sum of two waves with different wavelengths and amplitudes. The table below (and continued on the next page) lists the coordinates of points on the two waves.

1. Use coordinates on the table and the graph paper to graph wave 1 and wave 2 individually. Connect each set of points with a smooth curve that looks like a wave. Then, answer questions 2 – 9.

2. What is the amplitude of wave 1?

3. What is the amplitude of wave 2?

4. What is the wavelength of wave 1?

5. What is the wavelength of wave 2?

6. How many wavelengths of wave 1 did you draw?

7. How many wavelength of wave 2 did you draw?

8. Use the superposition principle to find the wave that results from the interference of the two waves.
   a. To do this, simply add the heights of wave 1 and wave 2 at each point and record the values in the last column. The first four points are done for you.
   b. Then use the points in last column to graph the new wave. Connect the points with a smooth curve. You should see a pattern that combines the two original waves.

9. Describe the wave created by adding the two original waves.

<table>
<thead>
<tr>
<th>$x$-axis (blocks)</th>
<th>Height of wave 1 ($y$-axis blocks)</th>
<th>Height of wave 2 ($y$-axis blocks)</th>
<th>Height of wave 1 + wave 2 ($y$-axis blocks)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>0.8</td>
<td>2</td>
<td>2.8</td>
</tr>
<tr>
<td>2</td>
<td>1.5</td>
<td>0</td>
<td>1.5</td>
</tr>
<tr>
<td>3</td>
<td>2.2</td>
<td>-2</td>
<td>0.2</td>
</tr>
<tr>
<td>4</td>
<td>2.8</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>$x$-axis (blocks)</td>
<td>Height of wave 1 (y-axis blocks)</td>
<td>Height of wave 2 (y-axis blocks)</td>
<td>Height of wave 1 + wave 2 (y-axis blocks)</td>
</tr>
<tr>
<td>------------------</td>
<td>---------------------------------</td>
<td>---------------------------------</td>
<td>---------------------------------</td>
</tr>
<tr>
<td>5</td>
<td>3.3</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>3.7</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>3.9</td>
<td>-2</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>4</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>3.9</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>3.7</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>3.3</td>
<td>-2</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>2.8</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>13</td>
<td>2.2</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>14</td>
<td>1.5</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>0.8</td>
<td>-2</td>
<td></td>
</tr>
<tr>
<td>16</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>17</td>
<td>-0.8</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>18</td>
<td>-1.5</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>19</td>
<td>-2.2</td>
<td>-2</td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>-2.8</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>21</td>
<td>-3.3</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>22</td>
<td>-3.7</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>23</td>
<td>-3.9</td>
<td>-2</td>
<td></td>
</tr>
<tr>
<td>24</td>
<td>-4</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>25</td>
<td>-3.9</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>26</td>
<td>-3.7</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>27</td>
<td>-3.3</td>
<td>-2</td>
<td></td>
</tr>
<tr>
<td>28</td>
<td>-2.8</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>29</td>
<td>-2.2</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>30</td>
<td>-1.5</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>31</td>
<td>-0.8</td>
<td>-2</td>
<td></td>
</tr>
<tr>
<td>32</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
</tbody>
</table>
The loudness of sound is measured in decibels (dB). Most sounds fall between zero and 100 on the decibel scale making it a very convenient scale to understand and use. Each increase of 20 decibels (dB) for a sound will be about twice as loud to your ears. Use the following table to help you answer the questions.

<table>
<thead>
<tr>
<th>Decibel Level</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>10-15 dB</td>
<td>A quiet whisper 3 feet away</td>
</tr>
<tr>
<td>30-40 dB</td>
<td>Background noise in a house</td>
</tr>
<tr>
<td>65 dB</td>
<td>Ordinary conversation 3 feet away</td>
</tr>
<tr>
<td>70 dB</td>
<td>City traffic</td>
</tr>
<tr>
<td>90 dB</td>
<td>A jackhammer cutting up the street 10 feet away</td>
</tr>
<tr>
<td>100 dB</td>
<td>Listening to headphones at maximum volume</td>
</tr>
<tr>
<td>110 dB</td>
<td>Front row at a rock concert</td>
</tr>
<tr>
<td>120 dB</td>
<td>The threshold of physical pain from loudness</td>
</tr>
</tbody>
</table>

**Example**

**How many decibels would a sound have if its loudness was twice that of city traffic?**

**Given**

From the table, the loudness of city traffic is 70 dB.

**Looking for**

The decibel reading for a sound twice as loud as traffic.

**Solution**

City traffic = 70 dB
Adding 20 dB doubles the loudness.
70 dB + 20 dB = 90 dB
90 dB is twice as loud as city traffic.

**Practice**

1. How many times louder than a jackhammer does the front row at a rock concert sound?
2. How many decibels would you hear in a room that sounds twice as loud as an average (35 dB) house?
3. You have your headphones turned all the way up.
   a. If you want them to sound half as loud, to what decibel level must the music be set?
   b. If you want them to sound 1/4 as loud, to what decibel level must the music be set?
4. How many times louder than city traffic does the front row at a rock concert sound?
5. When you whisper, you produce a 10-dB sound.
   a. When you speak quietly, your voice sounds twice as loud as a whisper. How many decibels is this?
   b. When you speak normally, your voice sounds 4 times as loud as a whisper. How many decibels is this?
   c. When you yell, your voice sounds 8 times as loud as a whisper. How many decibels is this?
Light is a form of energy. *Light intensity* describes the amount of energy per second falling on a surface, using units of watts per meter squared (W/m²). Light intensity follows an inverse square law. This means that the intensity decreases as the square of the distance from the source. For example, if you double the distance from the source, the light intensity is one-fourth its original value. If you triple the distance, the light intensity is one-ninth its original value.

Most light sources distribute their light equally in all directions, producing a spherical pattern. The area of a sphere is $4\pi r^2$, where $r$ is the radius or the distance from the light source. For a light source, the intensity is the power per area. The light intensity equation is:

$$I = \frac{P}{A} = \frac{P}{4\pi r^2}$$

Remember that the power in this equation is the amount of light emitted by the light source. When you think of a “100 watt” light bulb, the number of watts represents how much energy the light bulb uses, not how much light it emits. Most of the energy in an incandescent light bulb is emitted as heat, not light. That 100-watt light bulb may emit less than 1 watt of light energy with the rest being lost as heat.

Solve the following problems using the intensity equation. The first problem is done for you.

1. For a light source of 60 watts, what is the intensity of light 1 meter away from the source?

$$I = \frac{P}{A} = \frac{60 \text{ W}}{4\pi (1 \text{ m})^2} = 4.8 \text{ W/m}^2$$

2. For a light source of 60 watts, what is the intensity of light 10 meters away from the source?

3. For a light source of 60 watts, what is the intensity of light 20 meters away from the source?

4. If the distance from a light source doubles, how does light intensity change?

5. Answer the following problems for a distance of 4 meters from the different light sources.
   a. What is the intensity of light 4 meters away from a 1-watt light source?
   b. What is the intensity of light 4 meters away from a 10-watt light source?
   c. What is the intensity of light 4 meters away from a 100-watt light source?
   d. What is the intensity of light 4 meters away from a 1,000-watt light source?

6. What is the relationship between the watts of a light source and light intensity?
The Law of Reflection

The law of reflection works perfectly with light and the smooth surface of a mirror. However, you can apply this law to other situations. It can help you win a game of pool or pass a basketball to a friend on the court.

In this skill sheet you will review the law of reflection and perform practice problems that utilize this law. Use a protractor to make your angles correct in your diagrams.

The law of reflection states that when an object hits a surface, its angle of incidence will equal the angle of reflection. This is true when the object is light and the surface is a flat, smooth mirror. When the object and the surface are larger and lack smooth surfaces (like a basketball and a gym floor), the angles of incidence and reflection are nearly but not always exactly equal. The angles are close enough that understanding the law of reflection can help you improve your game.

A light ray strikes a flat mirror with a 30-degree angle of incidence. Draw a ray diagram to show how the light ray interacts with the mirror. Label the normal line, the incident ray, and the reflected ray.

Solution:

1. When we talk about angles of incidence and reflection, we often talk about the normal. The normal to a surface is an imaginary line that is perpendicular to the surface. The normal line starts where the incident ray strikes the mirror. A normal line is drawn for you in the sample problem above.
   a. Draw a diagram that shows a mirror with a normal line and a ray of light hitting the mirror at an angle of incidence of 60 degrees.
   b. In the diagram above, label the angle of reflection. How many degrees is this angle of reflection?
2. Light strikes a mirror’s surface at 20 degrees to the normal. What will the angle of reflection be?

3. A ray of light strikes a mirror. The angle formed by the incident ray and the reflected ray measures 90 degrees. What are the measurements of the angle of incidence and the angle of reflection?

4. In a game of basketball, the ball is bounced (with no spin) toward a player at an angle of 40 degrees to the normal. What will the angle of reflection be? Draw a diagram that shows this play. Label the angles of incidence and reflection and the normal.

Use a protractor to figure out the angles of incidence and reflection for the following problems.

5. Because a lot of her opponent’s balls are in the way for a straight shot, Amy is planning to hit the cue ball off the side of the pool table so that it will hit the 8-ball into the corner pocket. In the diagram, show the angles of incidence and reflection for the path of the cue ball. How many degrees does each angle measure?

6. You and a friend are playing pool. You are playing solids and he is playing stripes. You have one ball left before you can try for the eight ball. Stripe balls are in the way. You plan on hitting the cue ball behind one of the stripe balls so that it will hit the solid ball and force it to follow the pathway shown in the diagram. Use your protractor to figure out what angles of incidence and reflection are needed at points A and B to get the solid ball into the far side pocket.
Refraction

When light rays cross from one material to another they bend. This bending is called refraction. Refraction is a useful phenomenon. All kinds of optics, from glasses to camera lenses to binoculars depend on refraction.

If you are standing on the shore looking at a fish in a stream, the fish appears to be in a slightly different place than it really is. That’s because light rays that bounce off the fish are refracted at the boundary between water and air. If you are a hunter trying to spear this fish, you better know about this phenomenon or the fish will get away.

Why do the light rays bend as they cross from water into air?

A light ray bends because light travels at different speeds in different materials. In a vacuum, light travels at a speed of $3 \times 10^8$ m/sec. But when light travels through a material, it is absorbed and re-emitted by each atom or molecule it hits. This process of absorption and emission slows the light ray’s speed. We experience this slowdown as a bend in the light ray. The greater the difference in the light ray’s speed through two different materials, the greater the bend in the path of the ray.

The index of refraction ($n$) for a material is the ratio of the speed of light in a vacuum to the speed of light in the material.

\[
\text{Index of refraction} = \frac{\text{speed of light in a vacuum}}{\text{speed of light in a material}}
\]

The index of refraction for some common materials is given below:

<table>
<thead>
<tr>
<th>Material</th>
<th>Index of refraction ($n$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vacuum</td>
<td>1.0</td>
</tr>
<tr>
<td>Air</td>
<td>1.0001</td>
</tr>
<tr>
<td>Water</td>
<td>1.33</td>
</tr>
<tr>
<td>Glass</td>
<td>1.5</td>
</tr>
<tr>
<td>Diamond</td>
<td>2.42</td>
</tr>
</tbody>
</table>

**PRACTICE 1**

1. Could the index of refraction for a material ever be less than 1.0? Explain.

2. Explain why the index of refraction for air (a gas) is smaller than the index of refraction for a solid like glass.

3. Calculate the speed of light in water, glass, and diamond using the index of refraction and the speed of light in a vacuum ($3 \times 10^8$ m/sec).

4. When a light ray moves from water into air, does it slow down or speed up?

5. When a light ray moves from water into glass, does it slow down or speed up?
**Which way does the light ray bend?**

Now let’s look at some ray diagrams showing refraction. To make a refraction ray diagram, draw a solid line to show the boundary between the two materials (water and air in this case). Arrows are used to represent the incident and refracted light rays. The normal is a dashed line drawn perpendicular to the boundary between the surfaces. It starts at the point where the incident ray hits the boundary.

As you can see, the light ray is bent toward the normal as it crosses from air into water. Light rays always bend toward the normal when they move from a low-$n$ to a high-$n$ material. The opposite occurs when light rays travel from a high-$n$ to a low-$n$ material. These light rays bend away from the normal.

The amount of bending that occurs depends on the difference in the index of refraction of the two materials. A large difference in $n$ causes a greater bend than a small difference.

**PRACTICE 2**

1. A light ray moves from water ($n = 1.33$) to a transparent plastic (polystyrene $n = 1.59$). Will the light ray bend toward or away from the normal?

2. A light ray moves from sapphire ($n = 1.77$) to air ($n = 1.0001$). Does the light ray bend toward or away from the normal?

3. Which light ray will be bent more, one moving from diamond ($n = 2.42$) to water ($n = 1.33$), or a ray moving from sapphire ($n = 1.77$) to air ($n = 1.0001$)?

4. The diagrams below show light traveling from water (A) into another material (B). Using the chart above, label material B for each diagram as helium, water, emerald, or cubic zirconia.

<table>
<thead>
<tr>
<th>Material</th>
<th>Index of refraction ($n$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Helium</td>
<td>1.00004</td>
</tr>
<tr>
<td>Water</td>
<td>1.33</td>
</tr>
<tr>
<td>Emerald</td>
<td>1.58</td>
</tr>
<tr>
<td>Cubic Zirconia</td>
<td>2.17</td>
</tr>
</tbody>
</table>
Ray Diagrams

This skill sheet gives you some practice making ray diagrams. A ray diagram helps you determine where an image produced by a lens will form and shows whether the image is upside down or right side up. For each question on this skill sheet, read the directions carefully and plot your ray diagram in the space provided.

1. Of the diagrams below, which one correctly illustrates how light rays come off an object? Explain your answer.

   - All directions
   - Only one direction
   - All directions, corkscrew pathways

   ![Diagram Options]

2. Of the diagrams below, which one correctly illustrates how a light ray enters and exits a piece of thick glass? Explain your answer.

   - Light through thick glass

   ![Diagram Options]

   In your own words, explain what happens to light as it enters glass from the air. Why does this happen? Use the terms *refraction* and *index of refraction* in your answer.

3. Of the diagrams below, which one correctly illustrates how parallel light rays enter and exit a converging lens? Explain your answer.

   ![Diagram Options]

4. Draw a diagram of a converging lens that has a focal point of 10 centimeters. In your diagram, show three parallel lines entering the lens and exiting the lens. Show the light rays passing through the focal point of the lens. Be detailed in your diagram and provide labels.
A ray diagram helps you see where the image produced by a lens appears. The components of the diagram include the lens, the principal axis, the focal point, the object, and three lines drawn from the tip of the object and through the lens. These light rays meet at a point and intersect on the other side of the lens. Where the light rays meet indicates where the image of the object appears.

**Example**

A lens has a focal length of 2 centimeters. An object is placed 4 centimeters to the left of the lens. Follow the steps to make a ray diagram using this information. Trace the rays and predict where the image will form.

**Steps:**

- Draw a lens and show the principal axis.
- Draw a line that shows the plane of the lens.
- Make a dot at the focal point of the lens on the right and left sides of the lens.
- Place an arrow (pointing upward and perpendicular to the principle axis) at 4 centimeters on the left side of the lens.
- **Line 1:** Draw a line from the tip of the arrow that is parallel to the principal axis on the left, and that goes through the focal point on the right of the lens.
- **Line 2:** Draw a line from the tip of the arrow that goes through the center of the lens (where the plane and the principal axis cross).
- **Line 3:** Draw a line from the tip of the arrow that goes through the focal point on the left side of the lens, through the lens, and parallel to the principal axis on the right side of the lens.
- Lines 1, 2, and 3 converge on the right side of the lens where the tip of the image of the arrow appears.
- The image is upside down compared with the object.
1. A lens has a focal length of 4 centimeters. An object is placed 8 centimeters to the left of the lens. Trace the rays and predict where the image will form. Is the image bigger, smaller, or inverted as compared with the object?

2. **Challenge question:** An arrow is placed at 3 centimeters to the left of a converging lens. The image appears at 3 centimeters to the right of the lens. What is the focal length of this lens? (HINT: Place a dot to the right of the lens where the image of the tip of the arrow will appear. You will only be able to draw lines 1 and 2. Where does line 1 cross the principal axis if the image appears at 3 centimeters?)

3. What happens when an object is placed at a distance from the lens that is less than the focal length? Use the term *virtual image* in your answer.
Radio waves, microwaves, visible light, and x-rays are familiar kinds of electromagnetic waves. All of these waves have characteristic wavelengths and frequencies. Wavelength is measured in meters. It describes the length of one complete oscillation. Frequency describes the number of complete oscillations per second. It is measured in hertz, which is another way of saying “cycles per second.” The higher the wave’s frequency, the more energy it carries.

Frequency, wavelength, and speed

In a vacuum, all electromagnetic waves travel at the same speed: \(3.0 \times 10^8 \text{ m/sec}\). This quantity is often called “the speed of light” but it really refers to the speed of all electromagnetic waves, not just visible light. It is such an important quantity in physics that it has its own symbol, \(c\).

The speed of light is related to frequency \(f\) and wavelength \(\lambda\) by the formula to the right.

The different colors of light that we see correspond to different frequencies. The frequency of red light is lower than the frequency of blue light. Because the speed of both kinds of light is the same, a lower frequency wave has a longer wavelength. A higher frequency wave has a shorter wavelength. Therefore, red light’s wavelength is longer than blue light’s.

When we know the frequency of light, the wavelength is given by: \(\lambda = \frac{c}{f}\)

When we know the wavelength of light, the frequency is given by: \(f = \frac{c}{\lambda}\)
Answer the following problems and show your work.

1. Yellow light has a longer wavelength than green light. Which color of light has the higher frequency?

2. Green light has a lower frequency than blue light. Which color of light has a longer wavelength?

3. Calculate the wavelength of violet light with a frequency of \(750 \times 10^{12}\) Hz.

4. Calculate the frequency of yellow light with a wavelength of \(580 \times 10^{-9}\) m.

5. Calculate the wavelength of red light with a frequency of \(460 \times 10^{12}\) Hz.

6. Calculate the frequency of green light with a wavelength of \(530 \times 10^{-9}\) m.

7. One light beam has wavelength, \(\lambda_1\), and frequency, \(f_1\). Another light beam has wavelength, \(\lambda_2\), and frequency, \(f_2\). Write a proportion that shows how the ratio of the wavelengths of these two light beams is related to the ratio of their frequencies.

8. The waves used by a microwave oven to cook food have a frequency of 2.45 gigahertz (2.45\( \times 10^9\) Hz). Calculate the wavelength of this type of wave.

9. A radio station has a frequency of 90.9 megahertz (9.09\( \times 10^7\) Hz). What is the wavelength of the radio waves the station emits from its radio tower?

10. An x-ray has a wavelength of 5 nanometers (5.0\( \times 10^{-9}\) m). What is the frequency of x-rays?

11. The ultraviolet rays that cause sunburn are called UV-B rays. They have a wavelength of approximately 300 nanometers (3.0\( \times 10^{-7}\) m). What is the frequency of a UV-B ray?

12. Infrared waves from the sun are what make our skin feel warm on a sunny day. If an infrared wave has a frequency of 3.0\( \times 10^{12}\) Hz, what is its wavelength?

13. Electromagnetic waves with the highest amount of energy are called gamma rays. Gamma rays have wavelengths of less than 10-trillionths of a meter (1.0\( \times 10^{-11}\) m).

   a. Determine the frequency that corresponds with this wavelength.

   b. Is this the minimum or maximum frequency of a gamma ray?

14. Use the information from this sheet to order the following types of waves from lowest to highest frequency: visible light, gamma rays, x-rays, infrared waves, ultraviolet rays, microwaves, and radio waves.

15. Use the information from this sheet to order the following types of waves from shortest to longest wavelength: visible light, gamma rays, x-rays, infrared waves, ultraviolet rays, microwaves, and radio waves.
Doppler Shift

You learned about Doppler shift as it relates to sound in Unit 7. The Doppler shift is also an important tool used by astronomers to study the motion of objects, such as stars and galaxies, in space. For example, if an object is moving toward Earth, the light waves it emits are compressed, shifting them toward the blue end (shorter wavelengths, higher frequencies) of the visible spectrum. If an object is moving away from Earth, the light waves it emits are stretched, shifting them toward the red end (longer wavelengths, lower frequencies) of the visible spectrum. In this skill sheet, you will practice solving problems that involve light and doppler shift.

Understanding Doppler shift

Astronomers use a spectrometer to determine which elements are found in stars and other objects in space. When burned, each element on the periodic table produces a characteristic set of spectral lines. When an object in space is moving very fast, its spectral lines show the characteristic patterns for the elements it contains. However, these lines are shifted.

If the object is moving away from Earth, its spectral lines are shifted toward the red end of the spectrum to a longer wavelength. If the object is moving toward Earth, its spectral lines are shifted toward the blue end of the spectrum to a shorter wavelength.

By analyzing the shift in wavelength, you can also determine the speed at which a star is moving. The faster a star is moving, the larger the shift in wavelength. The following proportion is used to help you calculate the speed of a moving star. The speed of light is a constant value equal to $3 \times 10^8$ m/sec. The first problem is done for you.

The spectral lines emitted by a distant galaxy are analyzed. One of the lines for hydrogen has shifted from 450 nm to 498 nm. Is this galaxy moving away from or toward Earth? What is the speed of galaxy?

The galaxy is moving away from Earth at a speed of 33 million meters per second.
1. One of the spectral lines for a star has shifted from its stationary value of 535 nm to 545 nm.
   a. What is the difference in wavelength?
   b. What is the speed of this star?
   c. Is the star moving away from or toward Earth?

2. One of the spectral lines for a star has shifted from its stationary value of 560 nm to 544 nm.
   a. What is the difference in wavelength?
   b. What is the speed of this star?
   c. Is the star moving away from or toward Earth?

3. An astronomer has determined that two galaxies are moving away from Earth. A spectral line for galaxy A is red shifted from 501 nm to 510 nm. The same line for galaxy B is red shifted from 525 nm to 540 nm. Which galaxy is moving the fastest? Justify your answer.

4. Does the fact that both galaxies in the question above are moving away from Earth support or refute the Big Bang theory? Explain your answer.

5. The graphic to the right shows two spectral lines from an object that is not moving. Use an arrow to indicate the direction that the spectrum would appear to shift if the object was moving toward you.

6. The graphic to the right shows the spectral lines emitted by four moving objects. The spectral lines for when the object is stationary are shown as dotted lines on each spectrum. The faster a star is moving, the greater the shift in wavelength. Use the graphic to help you answer the following questions.
   a. Which of the spectra show an object that is moving toward you?
   b. Which of the spectra show an object that is moving away from you?
   c. Which of the spectra show an object that is moving the fastest away from you?
   d. Which of the spectra show an object that is moving the fastest toward you?

7. A star is moving away from Earth at $7 \times 10^6$ m/sec.
   a. The stationary wavelength of a spectral line is 450 nm. What is the difference in wavelength between the stationary and shifted line?
   b. Is the spectral line be shifted to a shorter or longer wavelength?
   c. What is the wavelength of the shifted line?
1.1 Scientific Processes

1. Maria and Elena’s question is: Does hot water in an ice cube tray freeze faster than cold water in an ice cube tray?

2. Maria’s hypothesis: Hot water will take longer to freeze into solid ice cubes than cold water, because the hot water molecules have to slow down more than cold water molecules to enter the solid state and become ice.

3. Examples of variables include:
   - Amount of water in each ice cube tray “slot” must be uniform.
   - Each ice cube tray must be made of same material, “slots” in all trays must be identical.
   - Placement of trays in freezer must provide equal cooling.
   - All “hot” water must be at the same initial temperature.
   - All “cold” water must be at the same initial temperature.

4. Examples of measurements include:
   - Initial temperature of hot water.
   - Initial temperature of cold water.
   - Time taken for water to freeze solid.

5. Sample procedure in 9 steps:
   1. Place 1 liter of water in a refrigerator to chill for 1 hour.
   2. Boil water in pot on a stove (water will be 100°C).
   3. Using pot holders, a kitchen funnel, and a medicine-measuring cup, carefully measure out 15 mL of boiling water into each slot in two, labeled, ice cube trays.
   4. Remove chilled water from refrigerator, measure temperature.
   5. Carefully measure 15 mL chilled water into each slot in two labeled, ice cube trays.
   6. Place trays on bottom shelf of freezer, along the back wall.
   7. Start timer.
   8. After 1/2 hour, begin checking trays every 15 minutes to see if solid ice has formed in any tray.
   9. Stop timing when at least one tray has solid ice cubes in it.

6. Repeating experiments ensures the accuracy of your results. Each time you are able to repeat your results, you reduce the effect of sources of error in the experiment that may come from following a certain procedure, human error, or from the conditions in which the experiment is taking place.

7. The only valid conclusion that can be drawn is (d). Although (c) is a true statement, this conclusion cannot be reached from this experiment alone.

8. Maria and Elena could ask a few of their friends to repeat their experiment. This would mean that the experiment would be repeated in other places with other freezers. If their friends are able to repeat the girls’ results, then the kind of freezer used can be eliminated as a factor that influenced the results.

9. A new question could be: Do dissolved minerals in water affect how fast water freezes?

For further study: Ask student to come up with a plan to test the validity of statements b and c. Encourage your students to research methods for measuring dissolved minerals and oxygen in water. Simple forms of these methods are described in unit 8, chapter 24.

1.2 Dimensional Analysis

1. Answers are:
   a. 39.4 inches
   b. 3.79 liters
   c. 0.624 miles
   d. 0.001 kilogram

2. 2400 eggs
3. 42.2 km
4. 105 km
5. 65.93 pounds
6. Answers are:
   a. 81 mph
   b. 37 mph
7. 154 pounds

1.2 International System of Measurements

1. 1,000 milligrams = 1 gram
2. 100,000 centimeters = 1 kilometer
3. 1,000,000 microliters = 1 liter
4. 1,000,000,000 nanoseconds = 1 second
5. 1,000,000,000 micrograms = 1 kilogram
6. 1,000,000,000 milliliters = 1 megaliter
7. 100 times larger
8. 1,000 times smaller
9. About 3 million times bigger
10. 1,000 times larger
11. Nanometer
12. Kilometer
13. Liter
14. Microgram
15. Nanosecond
1.2 Making Line Graphs

1. Answers are:

<table>
<thead>
<tr>
<th>Data pair not necessarily in order</th>
<th>Independent</th>
<th>Dependent</th>
</tr>
</thead>
<tbody>
<tr>
<td>Temp.</td>
<td>Hours of heating</td>
<td>Hours of heating</td>
</tr>
<tr>
<td>Stopping distance</td>
<td>Speed of a car</td>
<td>Speed of a car</td>
</tr>
<tr>
<td>Number of people in family</td>
<td>Cost per week for groceries</td>
<td>Number of people in family</td>
</tr>
<tr>
<td>Stream flow</td>
<td>Rainfall</td>
<td>Amount of rainfall</td>
</tr>
<tr>
<td>Tree age</td>
<td>Average tree height</td>
<td>Tree age</td>
</tr>
<tr>
<td>Test score</td>
<td>Number of hours studying for a test</td>
<td>Number of hours studying</td>
</tr>
<tr>
<td>Population of a city</td>
<td>Number of schools needed</td>
<td>Population of a city</td>
</tr>
</tbody>
</table>

2. Answers are:

<table>
<thead>
<tr>
<th>Lowest value</th>
<th>Highest value</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>28</td>
<td>28</td>
</tr>
<tr>
<td>10</td>
<td>87</td>
<td>77</td>
</tr>
<tr>
<td>0</td>
<td>4.2</td>
<td>4.2</td>
</tr>
<tr>
<td>-5</td>
<td>23</td>
<td>28</td>
</tr>
<tr>
<td>0</td>
<td>113</td>
<td>113</td>
</tr>
<tr>
<td>100</td>
<td>1,250</td>
<td>1,150</td>
</tr>
</tbody>
</table>

3. Answers are:

<table>
<thead>
<tr>
<th>Range</th>
<th>Number of lines</th>
<th>Range - No. of lines</th>
<th>Calculated scale (per line)</th>
<th>Adj. scale (per line)</th>
</tr>
</thead>
<tbody>
<tr>
<td>13</td>
<td>24</td>
<td>13 ÷ 24</td>
<td>0.54</td>
<td>1</td>
</tr>
<tr>
<td>83</td>
<td>43</td>
<td>83 ÷ 43</td>
<td>1.9</td>
<td>2</td>
</tr>
<tr>
<td>31</td>
<td>35</td>
<td>31 ÷ 35</td>
<td>0.88</td>
<td>1</td>
</tr>
<tr>
<td>100</td>
<td>33</td>
<td>100 ÷ 33</td>
<td>3.03</td>
<td>5</td>
</tr>
<tr>
<td>300</td>
<td>20</td>
<td>300 ÷ 20</td>
<td>15</td>
<td>15</td>
</tr>
<tr>
<td>900</td>
<td>15</td>
<td>900 ÷ 15</td>
<td>60</td>
<td>60</td>
</tr>
</tbody>
</table>

4. Answers are:

1.7 km/hr
2. 55 mph
3. 4.5 seconds
4. 5.9 hours; 490 mph
5. 4.0 km
6. 2.5 miles
7. 4.5 meters

1.3 Speed Problems

1. 17 km/hr
2. 55 mph
3. 4.5 seconds
4. 5.9 hours; 490 mph
5. 4.0 km
6. 2.5 miles
7. 4.5 meters

1.2 Making Line Graphs

1. Answers are:

<table>
<thead>
<tr>
<th>Data pair not necessarily in order</th>
<th>Independent</th>
<th>Dependent</th>
</tr>
</thead>
<tbody>
<tr>
<td>Temp.</td>
<td>Hours of heating</td>
<td>Hours of heating</td>
</tr>
<tr>
<td>Stopping distance</td>
<td>Speed of a car</td>
<td>Speed of a car</td>
</tr>
<tr>
<td>Number of people in family</td>
<td>Cost per week for groceries</td>
<td>Number of people in family</td>
</tr>
<tr>
<td>Stream flow</td>
<td>Rainfall</td>
<td>Amount of rainfall</td>
</tr>
<tr>
<td>Tree age</td>
<td>Average tree height</td>
<td>Tree age</td>
</tr>
<tr>
<td>Test score</td>
<td>Number of hours studying for a test</td>
<td>Number of hours studying</td>
</tr>
<tr>
<td>Population of a city</td>
<td>Number of schools needed</td>
<td>Population of a city</td>
</tr>
</tbody>
</table>

2. Answers are:

<table>
<thead>
<tr>
<th>Lowest value</th>
<th>Highest value</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>28</td>
<td>28</td>
</tr>
<tr>
<td>10</td>
<td>87</td>
<td>77</td>
</tr>
<tr>
<td>0</td>
<td>4.2</td>
<td>4.2</td>
</tr>
<tr>
<td>-5</td>
<td>23</td>
<td>28</td>
</tr>
<tr>
<td>0</td>
<td>113</td>
<td>113</td>
</tr>
<tr>
<td>100</td>
<td>1,250</td>
<td>1,150</td>
</tr>
</tbody>
</table>

3. Answers are:

<table>
<thead>
<tr>
<th>Range</th>
<th>Number of lines</th>
<th>Range - No. of lines</th>
<th>Calculated scale (per line)</th>
<th>Adj. scale (per line)</th>
</tr>
</thead>
<tbody>
<tr>
<td>13</td>
<td>24</td>
<td>13 ÷ 24</td>
<td>0.54</td>
<td>1</td>
</tr>
<tr>
<td>83</td>
<td>43</td>
<td>83 ÷ 43</td>
<td>1.9</td>
<td>2</td>
</tr>
<tr>
<td>31</td>
<td>35</td>
<td>31 ÷ 35</td>
<td>0.88</td>
<td>1</td>
</tr>
<tr>
<td>100</td>
<td>33</td>
<td>100 ÷ 33</td>
<td>3.03</td>
<td>5</td>
</tr>
<tr>
<td>300</td>
<td>20</td>
<td>300 ÷ 20</td>
<td>15</td>
<td>15</td>
</tr>
<tr>
<td>900</td>
<td>15</td>
<td>900 ÷ 15</td>
<td>60</td>
<td>60</td>
</tr>
</tbody>
</table>

4. Answers are:

1.7 km/hr
2. 55 mph
3. 4.5 seconds
4. 5.9 hours; 490 mph
5. 4.0 km
6. 2.5 miles
7. 4.5 meters

1.3 Speed Problems

1. 17 km/hr
2. 55 mph
3. 4.5 seconds
4. 5.9 hours; 490 mph
5. 4.0 km
6. 2.5 miles
7. 4.5 meters

1.2 Making Line Graphs

1. Answers are:

<table>
<thead>
<tr>
<th>Data pair not necessarily in order</th>
<th>Independent</th>
<th>Dependent</th>
</tr>
</thead>
<tbody>
<tr>
<td>Temp.</td>
<td>Hours of heating</td>
<td>Hours of heating</td>
</tr>
<tr>
<td>Stopping distance</td>
<td>Speed of a car</td>
<td>Speed of a car</td>
</tr>
<tr>
<td>Number of people in family</td>
<td>Cost per week for groceries</td>
<td>Number of people in family</td>
</tr>
<tr>
<td>Stream flow</td>
<td>Rainfall</td>
<td>Amount of rainfall</td>
</tr>
<tr>
<td>Tree age</td>
<td>Average tree height</td>
<td>Tree age</td>
</tr>
<tr>
<td>Test score</td>
<td>Number of hours studying for a test</td>
<td>Number of hours studying</td>
</tr>
<tr>
<td>Population of a city</td>
<td>Number of schools needed</td>
<td>Population of a city</td>
</tr>
</tbody>
</table>

2. Answers are:

<table>
<thead>
<tr>
<th>Lowest value</th>
<th>Highest value</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>28</td>
<td>28</td>
</tr>
<tr>
<td>10</td>
<td>87</td>
<td>77</td>
</tr>
<tr>
<td>0</td>
<td>4.2</td>
<td>4.2</td>
</tr>
<tr>
<td>-5</td>
<td>23</td>
<td>28</td>
</tr>
<tr>
<td>0</td>
<td>113</td>
<td>113</td>
</tr>
<tr>
<td>100</td>
<td>1,250</td>
<td>1,150</td>
</tr>
</tbody>
</table>

3. Answers are:

<table>
<thead>
<tr>
<th>Range</th>
<th>Number of lines</th>
<th>Range - No. of lines</th>
<th>Calculated scale (per line)</th>
<th>Adj. scale (per line)</th>
</tr>
</thead>
<tbody>
<tr>
<td>13</td>
<td>24</td>
<td>13 ÷ 24</td>
<td>0.54</td>
<td>1</td>
</tr>
<tr>
<td>83</td>
<td>43</td>
<td>83 ÷ 43</td>
<td>1.9</td>
<td>2</td>
</tr>
<tr>
<td>31</td>
<td>35</td>
<td>31 ÷ 35</td>
<td>0.88</td>
<td>1</td>
</tr>
<tr>
<td>100</td>
<td>33</td>
<td>100 ÷ 33</td>
<td>3.03</td>
<td>5</td>
</tr>
<tr>
<td>300</td>
<td>20</td>
<td>300 ÷ 20</td>
<td>15</td>
<td>15</td>
</tr>
<tr>
<td>900</td>
<td>15</td>
<td>900 ÷ 15</td>
<td>60</td>
<td>60</td>
</tr>
</tbody>
</table>

4. Answers are:

1.7 km/hr
2. 55 mph
3. 4.5 seconds
4. 5.9 hours; 490 mph
5. 4.0 km
6. 2.5 miles
7. 4.5 meters

1.3 Speed Problems

1. 17 km/hr
2. 55 mph
3. 4.5 seconds
4. 5.9 hours; 490 mph
5. 4.0 km
6. 2.5 miles
7. 4.5 meters

1.2 Making Line Graphs

1. Answers are:

<table>
<thead>
<tr>
<th>Data pair not necessarily in order</th>
<th>Independent</th>
<th>Dependent</th>
</tr>
</thead>
<tbody>
<tr>
<td>Temp.</td>
<td>Hours of heating</td>
<td>Hours of heating</td>
</tr>
<tr>
<td>Stopping distance</td>
<td>Speed of a car</td>
<td>Speed of a car</td>
</tr>
<tr>
<td>Number of people in family</td>
<td>Cost per week for groceries</td>
<td>Number of people in family</td>
</tr>
<tr>
<td>Stream flow</td>
<td>Rainfall</td>
<td>Amount of rainfall</td>
</tr>
<tr>
<td>Tree age</td>
<td>Average tree height</td>
<td>Tree age</td>
</tr>
<tr>
<td>Test score</td>
<td>Number of hours studying for a test</td>
<td>Number of hours studying</td>
</tr>
<tr>
<td>Population of a city</td>
<td>Number of schools needed</td>
<td>Population of a city</td>
</tr>
</tbody>
</table>

2. Answers are:

<table>
<thead>
<tr>
<th>Lowest value</th>
<th>Highest value</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>28</td>
<td>28</td>
</tr>
<tr>
<td>10</td>
<td>87</td>
<td>77</td>
</tr>
<tr>
<td>0</td>
<td>4.2</td>
<td>4.2</td>
</tr>
<tr>
<td>-5</td>
<td>23</td>
<td>28</td>
</tr>
<tr>
<td>0</td>
<td>113</td>
<td>113</td>
</tr>
<tr>
<td>100</td>
<td>1,250</td>
<td>1,150</td>
</tr>
</tbody>
</table>

3. Answers are:

<table>
<thead>
<tr>
<th>Range</th>
<th>Number of lines</th>
<th>Range - No. of lines</th>
<th>Calculated scale (per line)</th>
<th>Adj. scale (per line)</th>
</tr>
</thead>
<tbody>
<tr>
<td>13</td>
<td>24</td>
<td>13 ÷ 24</td>
<td>0.54</td>
<td>1</td>
</tr>
<tr>
<td>83</td>
<td>43</td>
<td>83 ÷ 43</td>
<td>1.9</td>
<td>2</td>
</tr>
<tr>
<td>31</td>
<td>35</td>
<td>31 ÷ 35</td>
<td>0.88</td>
<td>1</td>
</tr>
<tr>
<td>100</td>
<td>33</td>
<td>100 ÷ 33</td>
<td>3.03</td>
<td>5</td>
</tr>
<tr>
<td>300</td>
<td>20</td>
<td>300 ÷ 20</td>
<td>15</td>
<td>15</td>
</tr>
<tr>
<td>900</td>
<td>15</td>
<td>900 ÷ 15</td>
<td>60</td>
<td>60</td>
</tr>
</tbody>
</table>

4. Answers are:

1.7 km/hr
2. 55 mph
3. 4.5 seconds
4. 5.9 hours; 490 mph
5. 4.0 km
6. 2.5 miles
7. 4.5 meters

1.3 Speed Problems

1. 17 km/hr
2. 55 mph
3. 4.5 seconds
4. 5.9 hours; 490 mph
5. 4.0 km
6. 2.5 miles
7. 4.5 meters
11. 1,200 meters
12. Answers are:
   a. 42 km
   b. 9.2 km/hr
13. Answers are:
   a. 0.2 km/min
   b. 0.5 km/min
   c. 0.3 km/min faster by bicycle
14. 12.5 km
15. 40 minutes
16. 90 km/hr

13. Working with Quantities and Rates

1. Answers are:
   a. 20 in²
   b. 24 eggs
   c. Cannot combine
   d. 7 cookies
   e. Cannot combine
   f. 10 (no units; they cancel)
2. $12/hour
3. 24 students/classroom
4. 600 meters/minute
5. \( \frac{150 \text{ blinks}}{25 \text{ clinks}} = \frac{6 \text{ blinks}}{\text{clink}} \)
6. cm
7. program
8. shrimp

13. Problem Solving with Rates

1. \( \frac{365 \text{ days}}{1 \text{ year}} \)
2. \( \frac{1 \text{ foot}}{12 \text{ inches}} \)
3. \( \frac{3 \text{ small pizzas}}{3 \text{ dollars}} \)
4. \( \frac{3 \text{ boxes}}{36 \text{ pencils}} \)
5. \( \frac{360 \text{ miles}}{18 \text{ gallons of gasoline}} \)
6. \( \frac{2,100 \text{ calories}}{1 \text{ year}} \)
7. \( \frac{1,095 \text{ sodas}}{1 \text{ week}} \)
8. \( \frac{725,760 \text{ heatbeats}}{1 \text{ week}} \)
9. \$27.48
10. 396 miles or 400 miles
11. \( \frac{6.6 \text{ miles}}{1 \text{ hour}} \)
12. \( 22.2 \text{ lbs} \)
13. \( 55 \text{ kg} \)
14. \( \frac{5.5 \text{ miles}}{1 \text{ hour}} \)
15. \$280
16. About 10 years
17. \( 53 \text{ grams} \)
18. \( 0.113 \frac{\text{miles}}{\text{hour}} \)
19. 270 pills
20. 95 feet/sec

2.1 Mass vs. Weight

1. 16 pounds
2. 2.6 pounds
3. 7.0 kilograms
4. If you stepped on a bathroom scale on the moon, the spring would be compressed one-sixth as much as it would on Earth. The dial would tell you that your weight was one-sixth of your Earth weight.
5. Yes, a balance would function correctly on the moon. The unknown mass would tip the balance one-sixth as far as it would on Earth, but the masses of known quantity would tip the balance one-sixth as far in the opposite direction as they did on Earth. The net result is that it would take the same amount of mass to equalize the balance on the moon as it did on Earth. (In the free fall environment of the space shuttle, however, the masses wouldn’t stay on the balance, so the balance would not work).

6. Answers are:
   a. As the elevator begins to accelerate upward, the scale reading is greater than the normal weight. As the elevator accelerates downward, the scale reads less than the normal weight.

2.2 Acceleration Problems

<table>
<thead>
<tr>
<th>Problem</th>
<th>Acceleration</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-0.75 m/sec^2</td>
</tr>
<tr>
<td>2</td>
<td>-8.9 m/sec^2</td>
</tr>
<tr>
<td>3</td>
<td>23 mph/sec</td>
</tr>
<tr>
<td>4</td>
<td>25 km/hr/sec</td>
</tr>
</tbody>
</table>

5. The cheetah. The cheetah’s acceleration in km/hr/sec is 37 km/hr/sec, which is 12 km/hr/sec faster than the car.

6. Answers are:

<table>
<thead>
<tr>
<th>Time (seconds)</th>
<th>Speed (km/h)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 (start)</td>
<td>0 (start)</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>4</td>
<td>6</td>
</tr>
<tr>
<td>6</td>
<td>9</td>
</tr>
<tr>
<td>8</td>
<td>12</td>
</tr>
<tr>
<td>10</td>
<td>15</td>
</tr>
</tbody>
</table>

The acceleration of the ball is 1.5 km/hr/sec.

2.2 Newton’s Second Law

<table>
<thead>
<tr>
<th>Problem</th>
<th>Force</th>
<th>Mass</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.1 m/sec^2</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>83 m/sec^2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>82 N</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>6 kg</td>
<td></td>
</tr>
</tbody>
</table>

5. 9800 N
6. 900 kg
7. 1.9 m/sec^2

2.3 Acceleration due to Gravity

<table>
<thead>
<tr>
<th>Problem</th>
<th>Velocity</th>
<th>Depth</th>
<th>Height</th>
<th>Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-14.7 m/sec</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>11.3 m/sec</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>-76.4 m/sec</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>-16 m/sec</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>86 meters</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>11 meters; yes</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

7. height = 5.9 meters (The maximum height would have occurred in half the total time that the serve was in the air or 1.1 seconds.)
8. time = 5.6 seconds
9. time = 7.0 seconds
10. Students use the equations to check their answers for the other questions.

2.4 Analyzing Graphs of Motion Without Numbers

1. Little Red Riding Hood. Graph Little Red Riding Hood:

   ![Graph of Position vs. Time](image1)
   ![Graph of Speed vs. Time](image2)

2. The Tortoise and the Hare. Use two lines to graph both the tortoise and the hare:

   ![Graph of Position vs. Time](image3)
   ![Graph of Speed vs. Time](image4)
3. The Skyrocket. Graph the altitude of the rocket:

- The line begins and ends on the baseline, therefore Tim must start from and return to his house.
- The line rises toward the first peak as a downward curved line that becomes horizontal. This indicates that Tim's pace toward Caroline's house slowed to a stop.
- Then the line rises steeply to the first peak. This indicates that after his stop, Tim continues toward Caroline's house faster than before.
- The first peak is sharp, indicating that Tim did not spend much time at Caroline's house on first arrival.
- The line then falls briefly, turns to the horizontal, and then rises to a second peak. This indicates that Tim left, paused, and then returned quickly to Caroline's house.
- The line then remains at the second peak for a long time, then drops steeply to the baseline. This indicates that after spending a long time at Caroline's house, Tim probably ran home.

4. Each student story will include elements that are controlled by the graphs and creative elements that facilitate the story. Only the graph-controlled elements are described here.

2.4 Analyzing Graphs of Motion With Numbers

1. Answers are:
   a. The bicycle trip through hilly country.
   b. A walk in the park.
   c. Up and down the supermarket aisles.

2. Answers are:
   a. The honey bee among the flowers.

2.4 Acceleration and Speed-Time Graphs

1. Acceleration = 5 miles/hour/hour or 5 miles/hour²
2. Acceleration = -2 meters/minute/minute or -2 meters/minute²
3. Acceleration = 0 feet/minute/minute or 0 feet/minute² or no acceleration
4. Answers are:
   Segment 1: Acceleration = 2 feet/second/second, or 2 feet/second²
   Segment 2: Acceleration = 0.67 feet/second/second, or 0.67 feet/second²
5. Distance = 1,400 meters
6. Distance = 700 meters
7. Distance = 75 kilometers
3.1 Applying Newton’s Laws of Motion

Table answers are:

<table>
<thead>
<tr>
<th>Newton’s laws of motion</th>
<th>Write the law here in your own words</th>
<th>Example of the law</th>
</tr>
</thead>
<tbody>
<tr>
<td>The first law</td>
<td>An object will continue moving in a straight line at constant speed unless acted upon by an outside force.</td>
<td>A seat belt in a car prevents you from continuing to move forward when your car suddenly stops. The seat belt provides the “outside force.”</td>
</tr>
<tr>
<td>The second law</td>
<td>The acceleration ($a$) of an object is directly proportional to the force ($F$) on an object and inversely proportional to its mass ($m$). The formula that represents this law is $a = \frac{F}{m}$.</td>
<td>A bowling ball and a basketball, if dropped from the same height at the same time, will fall to Earth in the same amount of time. The resistance of the heavier ball to being moved due to its inertia is balanced by the greater gravitational force on this ball.</td>
</tr>
<tr>
<td>The third law</td>
<td>For every action force there is an equal and opposite reaction force.</td>
<td>When you push on a wall, it pushes back on you.</td>
</tr>
</tbody>
</table>

1. The purse continues to move forward and fall off of the seat whenever the car comes to a stop. This is due to Newton’s first law of motion which states that objects will continue their motion unless acted upon by an outside force. In this case, the floor of the car is the stopping force for the purse.
2. Newton’s third law of motion states that forces come in action and reaction pairs. When a diver exerts a force downward on the diving board, the board exerts an equal and opposite force upward on the diver. The diver can use this force to allow himself to be catapulted into the air for a really dramatic dive or cannonball.
3. Newton’s second law
4. The correct answer is b. One newton of force equals 1 kilogram-meter/second$^2$. These units are combined in Newton’s second law of motion: $F = \text{mass} \times \text{acceleration}.$
5. $\frac{0.3 \text{ m}}{\text{sec}} = \frac{F}{65 \text{ kg}}$  \[ F = \frac{0.3 \text{ m}}{\text{sec}^2} \times 65 \text{ kg} = 19.5 \text{ kg} \times \frac{m}{\text{sec}^2} \]
6. $a = \frac{2 \text{ kg} \times \text{m}}{10 \text{ kg} \times \text{sec}^2} = \frac{2 \text{ m}}{10 \text{ kg} \times \text{sec}^2} = \frac{0.2 \text{ m}}{\text{sec}^2}$
7. The hand pushing on the ball is an action force the ball provides a push back as a reaction force. The ball then provides an action force on the floor and the floor pushes back in reaction. Another pair of forces occurs between your feet and the floor.

3.1 Momentum

1. Momentum = 2,000 kg $\times \frac{35 \text{ m}}{\text{sec}} = 70,000 \text{ kg} \times \frac{\text{m}}{\text{sec}}$
2. Momentum = 1,000 kg $\times \frac{35 \text{ m}}{\text{sec}} = 35,000 \text{ kg} \times \frac{\text{m}}{\text{sec}}$
3. \[8 \text{ kg} \times \text{speed} = 16 \text{ kg} \times \frac{\text{m}}{\text{sec}}, \text{ speed} = \frac{2 \text{ m}}{\text{sec}}\]
4. \[\text{mass} \times \frac{0.5 \text{ m}}{\text{sec}} = 0.25 \text{ kg} \times \frac{\text{m}}{\text{sec}}\]
5. \[40,000 \text{ kg} \times \frac{\text{m}}{\text{sec}}\]
6. \[29 \text{ m} \times \frac{\text{sec}}{\text{sec}}\]
7. The force required to stop the 8.0-kilogram at 0.2 m/sec is 0.16 N. The force required to stop the 4.0-kilogram ball at 1.0 m/sec is 0.4 N. The 4.0-kilogram ball requires more force to stop.
8. \[1,225 \text{ kg}\]
9. \[4.2 \text{ kg} \times \frac{\text{m}}{\text{sec}}\]
10. \[15 \text{ m} \times \frac{\text{sec}}{\text{sec}}\]
11. \[0.01 \text{ kg} \times \frac{\text{m}}{\text{sec}}\]

3.1 Impulse

1. Time = 1 second
2. Answers are:
   a. Change in momentum = 30 kg·m/sec
   b. The value for the impulse would equal the change in momentum: 30 kg·m/sec (or 30 N·sec).
   c. 40 N $\times$ (time) = 30 N·sec; time = 30 N·sec/40 N = 0.75 sec
3. Answers are:
   a. Impulse = 60,000 N·sec
   b. The value for change in momentum equals impulse, therefore the answer is 60,000 N·sec (or 60,000 kg·m/sec)
   c. Initial speed = 30 m/sec
4. Answers are:
   a. Change in momentum = 300 kg·m/sec
   b. 300 N
5. Final speed = 20 m/sec
6. 3,240 N [Note to teachers: Remember that the change of velocity of the batted ball is greater (94 m/sec) because it changes from a positive 44 m/sec to a negative 50 m/sec.]
7. 4.8 seconds
8. When taking off or landing, a net force must be applied to a person's body to accelerate them at the same rate as the accelerating airplane. When moving at constant speed, no net force is applied to a passenger's body.
9. In order to break wood with the hand, a large force is applied in a short time. If the hand bounces, an even greater impulse was to hit the piece of wood.
10. Because the barrel of a rifle is longer than a pistol's barrel, the bullet from the rifle is acted upon by the expanding gases of the exploding gunpowder for a longer time. This causes the bullet of the rifle to reach a higher speed.
11. If the force applied by the opponent's punch can be extended over a longer time, the force of the blow is reduced, minimizing the chances of a knockout punch being delivered to the boxer.
12. Newton's 2nd law expressed as an equation is $F = ma$; since acceleration ($a$) is $\Delta v / \Delta t$, the expression $F \Delta t = m \Delta v$. Multiplying both sides of the equation by $\Delta t$, the equation becomes $F \Delta t = m \Delta v$; the expression impulse equals the change in momentum.
13. Some activities that people do involve impacts that occur either on purpose or by accident. Soft objects extend the time over which the force of an impact is felt. This means that the force felt by the person is less than it would be if the impact time was short. In other words, given the momentum of an impact situation, it's important to increase time so that force is small.

### 3.1 Momentum Conservation

1. $p = mv$; $p = (10.0 \text{ kg} \cdot \text{m/sec}) / (1.5 \text{ m/sec}); m = 6.7 \text{ kg}$
2. $v = p/m; v = (1000 \text{ kg} \cdot \text{m/sec}) / (2.5 \text{ kg}); v = 400 \text{ m/sec}$
3. $p = mv$ (mass is conventionally expressed in kilograms)
   
   $p = (0.045 \text{ kg}) (75.0 \text{ m/sec})$
   
   $p = 3.38 \text{ kg} \cdot \text{m/sec}$

4. $p_{(\text{before firing})} = p_{(\text{after firing})}$
   
   $m_1 v_1 + m_2 v_2 = m_3 v_3 + m_4 v_4$
   
   $400 \text{ kg} (0 \text{ m/sec}) + 10 \text{ kg} (0 \text{ m/sec}) = 400 \text{ kg} (v_2) + 10 \text{ kg} (20 \text{ m/sec})$
   
   $0 = 400 \text{ kg} (v_2) + 200 \text{ kg} \cdot \text{m/sec}$
   
   $(v_2) = (-200 \text{ kg} \cdot \text{m/sec}) / 400 \text{ kg}$
   
   $(v_2) = -0.50 \text{ m/sec}$

5. $p_{(\text{before throwing})} = p_{(\text{after throwing})}$
   
   $m_1 v_1 + m_2 v_2 = m_3 v_3 + m_4 v_4$
   
   $0 = m_1 (0.05 \text{ m/sec}) + 0.5 \text{ kg} (10.0 \text{ m/sec})$

6. $p = mv + \Delta p; p = mv + F \Delta t$
   
   $p = (80 \text{ kg})(3.0 \text{ m/sec}) + (800 \text{ N})(0.30 \text{ sec})$
   
   $p = 480 \text{ kg} \cdot \text{m/sec}$
   
   $v = p/m; v = (480 \text{ kg} \cdot \text{m/sec}) / 80 \text{ kg}; v = 6.0 \text{ m/sec}$

7. Answers are:
   
   a. $p = mv; p = (2000 \text{ kg})(30 \text{ m/sec}); p = 60,000 \text{ kg} \cdot \text{m/sec}$
   
   b. $F \Delta t = m \Delta v$
      
      $F = (m \Delta v) / (\Delta t); F = (60,000 \text{ kg} \cdot \text{m/sec}) / (0.72 \text{ sec})$
      
      $F = 83,000 \text{ N}$

8. Answers are:
   
   a. $P = (# \text{ of people})(mv); P = (2.0 \times 10^9)(60 \text{ kg})(7.0 \text{ m/sec})$
      
      $p = 8.4 \times 10^{11} \text{ kg} \cdot \text{m/sec}$
   
   b. $P_{(\text{before jumping})} = P_{(\text{after jumping})}$
      
      $m_1 v_1 + m_2 v_2 = m_3 v_3 + m_4 v_4$
      
      $0 = 8.4 \times 10^{11} \text{ kg} \cdot \text{m/sec} + 5.98 \times 10^{24}(v_4)$;
      
      $(v_4) = (-8.4 \times 10^{11} \text{ kg} \cdot \text{m/sec}) / 5.98 \times 10^{24}$
      
      Earth moves beneath their feet at the speed,
      
      $v_1 = -1.4 \times 10^{-13} \text{ m/sec}$

9. Answers are:
   
   a. $p = mv; p = (60 \text{ kg})(6.00 \text{ m/sec}); p = 360 \text{ kg} \cdot \text{m/sec}$
   
   b. $v_{av} = (v_i + v_f) / 2; v_{av} = (6.00 \text{ m/sec} + 0 \text{ m/sec}) / 2 = 3.00 \text{ m/sec}$

10. Since the gun and bullet are stationary before being fired, the momentum of the system is zero. The "kick" of the gun is the momentum of the gun that is equal but opposite to that of the bullet maintaining the "zero" momentum of the system.

11. It means that momentum is transferred without loss.

### 3.2 Work

1. Work is force acting upon an object to move it a certain distance. In scientific terms, work occurs ONLY when the force is applied in the same direction as the movement.
2. Work is equal to force multiplied by distance.
3. Work can be represented in joules or newton-meters.

4. Answers are:
   
   a. No work done
   
   b. Work done
   
   c. No work done
   
   d. Work done
   
   e. Work done

5. 100 N-m or 100 joules
6. 180 N-m or 180 joules
7. 100,000 N-m or 100,000 joules
8. 50 N-m to lift the sled; no work is done to carry the sled
9. No work was done by the mouse. The force on the ant was upward, but the distance was horizontal.
10. 10,000 joules

7. Answers are:
   
   a. $0.125 \text{ meters}$
   
   b. 27 pounds

12. 2,500 N or 562 pounds
13. 1,500 N
14. 54 N-m or 54 joules
15. 225 N-m or 225 joules
16. 0.50 meters
17. Answers are:
   
   a. No work was done.
   
   b. 100 N-m or 100 joules
18. Answers are:
   
   a. No work is done
   
   b. 11 N-m or 11 joules
   
   c. 400 N-m or 400 joules (Henry did the most work.)
### 3.2 Potential and Kinetic Energy

1. 19.6 joules
2. 6 joules
3. 0.204 kilograms or 204 grams
4. Answers are:

<table>
<thead>
<tr>
<th>Shelf height (meters)</th>
<th>Potential energy (joules)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>5</td>
</tr>
<tr>
<td>1.5</td>
<td>7.5</td>
</tr>
<tr>
<td>2.0</td>
<td>10</td>
</tr>
</tbody>
</table>

5. Answers are:
   a. The 4-kilogram object had more kinetic energy while being lifted. The kinetic energy of the 2-kilogram object was 196 joules. The kinetic energy of the 4-kilogram object was 196 joules.
   b. The object with more mass—the 4-kilogram object—had greater potential energy (392 joules). The potential energy of the 2-kilogram object was 4 joules. The kinetic energy of the 4-kilogram object being lifted.

### 3.3 Collisions and Momentum Conservation

1. \( p = mv; p = (100 \text{ kg})(3.5 \text{ m/s}); p = 350 \text{ kg·m/s} \)
2. \( p = mv; p = (75.0 \text{ kg})(5.00 \text{ m/s}); p = 375 \text{ kg·m/s} \)
3. Answer:
   \[ P(\text{before coupling}) = P(\text{after coupling}) \]
   \[
   m_1v_1 + m_2v_2 = (m_1 + m_2)(v_{3+4})
   
   (2000 \text{ kg})(5 \text{ m/s}) + (6000 \text{ kg})(-3 \text{ m/s}) = (8000 \text{ kg})(v_{3+4})
   
   v_{3+4} = -1 \text{ m/s or 1 m/s west}
   
4. Answer:
   \[ P(\text{before collision}) = P(\text{after collision}) \]
   \[
   m_1v_1 + m_2v_2 = m_3v_3 + m_4v_4
   
   (4 \text{ kg})(8 \text{ m/s}) + (1 \text{ kg})(0 \text{ m/s}) = (4 \text{ kg})(4.8 \text{ m/s}) + (1 \text{ kg})(v_4)
   
   v_4 = (32 \text{ kg·m/s} - 19.2 \text{ kg·m/s})/(1 \text{ kg}); v_4 = 12.8 \text{ m/s}
   
5. Answer:
   \[ P(\text{before shooting}) = P(\text{after shooting}) \]
   \[
   m_1v_1 + m_2v_2 = (m_3 + m_4)(v_{3+4})
   
   (0.0010 \text{ kg})(30 \text{ m/s}) + (0.35 \text{ kg})(0 \text{ m/s}) = (0.351 \text{ kg})(v_{3+4})
   
   (v_{3+4}) = (0.050 \text{ kg·m/s})/(0.351 \text{ kg}); (v_{3+4}) = 0.14 \text{ m/s}
   
6. Answer:
   \[ P(\text{before tackle}) = P(\text{after tackle}) \]
   \[
   m_1v_1 + m_2v_2 = (m_3 + m_4)(v_{3+4})
   
   (70 \text{ kg})(7 \text{ m/s}) + (100 \text{ kg})(-6 \text{ m/s}) = (170 \text{ kg})(v_{3+4})
   
   (v_{3+4}) = (-490 \text{ kg·m/s} - 600 \text{ kg·m/s})/(170 \text{ kg})
   
   (v_{3+4}) = 0.64 \text{ m/s}
   
   Terry is moved backwards at a speed of 0.64 m/s while Jared holds on.
   
7. Answer:
   \[ P(\text{before hand}) = P(\text{after hand}) \]
   \[
   m_1v_1 + m_2v_2 = (m_3 + m_4)(v_{3+4})
   
   (50.0 \text{ kg})(7.30 \text{ m/s}) + (100. \text{ kg})(16 \text{ m/s}) = (150 \text{ kg})(v_{3+4})
   
   (v_{3+4}) = (350 \text{ kg·m/s} + 1600 \text{ kg·m/s})/150 \text{ kg}
   
   (v_{3+4}) = 13 \text{ m/s}

8. Answer:
   \[ P(\text{before jump}) = P(\text{after jump}) \]
   \[
   m_1v_1 + m_2v_2 = (m_3 + m_4)(v_{3+4})
   
   (520 \text{ kg})(13 \text{ m/s}) + (85 \text{ kg})(3 \text{ m/s}) = (605 \text{ kg})(v_{3+4})
   
   v_{3+4} = (7015 \text{ kg·m/s})/(605 \text{ kg})
   
   v_{3+4} = 11.6 \text{ m/s}

9. Answer:
   \[ P(\text{before jumping}) = P(\text{after jumping}) \]
   \[
   m_1v_1 + m_2v_2 + m_3v_3 = m_4v_4 + m_5v_5 + m_6v_6
   
   (45 \text{ kg})(1 \text{ m/s}) + (45 \text{ kg})(1 \text{ m/s}) + (70 \text{ kg})(1 \text{ m/s}) =
   
   (45 \text{ kg})(3 \text{ m/s}) + (45 \text{ kg})(4 \text{ m/s}) + (70 \text{ kg})(v_6)
   
   160 \text{ kg·m/s} = (45 \text{ kg})(7 \text{ m/s}) + (70 \text{ kg})(v_6)
   
   (v_6) = (205 \text{ kg·m/s})/(70 \text{ kg})
   
   (v_6) = 2.93 \text{ m/s}

10. Answers are:
    a. Answer:
        \[ P(\text{before toss}) = P(\text{after toss}) \]
        \[
        m_1v_1 + m_2v_2 = (m_3 + m_4)(v_{3+4})
        
        (0.10 \text{ kg})(v_1) + (0.10 \text{ kg})(0 \text{ m/s}) = (0.20 \text{ kg})(15 \text{ m/s});
        
        (v_1) = (0.30 \text{ kg·m/s})(0.1 \text{ kg})
        
        (v_1) = 30 \text{ m/s}
    
    b. Answer:
        \[ P(\text{before collision}) = P(\text{after collision}) \]
        \[
        m_1v_1 + m_2v_2 = m_3v_3 + m_4v_4
        
        (0.10 \text{ kg})(30 \text{ m/s}) + (0.10 \text{ kg})(0 \text{ m/s}) = (0.10 \text{ kg})(v_3) +
        
        (0.10 \text{ kg})(40 \text{ m/s})
        
        (v_3) = (3.0 \text{ kg·m/s} + 4 \text{ kg·m/s})/(0.10 \text{ kg})
        
        (v_3) = 70 \text{ m/s}

The block slides away at a much higher speed of \( v_3 = 70 \text{ m/s} \). The “bouncy ball, by rebounding, has experienced a greater change in momentum. The block will experience this change in momentum as well.
4.1 Work Done Against Gravity

1. 1,323 joules
2. 207,000 joules
3. 20 joules
4. 3 meters
5. 3,375 joules
6. 80 kilograms

4.1 Power

1. 250 watts
2. 50 watts
3. 1,200 watts
4. 1,500 watts
5. 741 watts
6. 720 watts
7. work = 500 joules; power = 33 watts
8. 1,800 seconds or 30 minutes
9. 2,160,000 joules
10. 2,500 watts
11. 90,000 joules
12. work = 1,500 joules; time = 60 seconds
13. force =25 newtons; power = 250 watts
14. distance = 100 meters; power = 1,000 watts
15. force = 333 newtons, work = 5,000 joules

4.2 Mechanical Advantage

1. 4
2. 0.4
3. 100 newtons
4. 25 newtons
5. 300 newtons
6. 26 newtons
7. 3
8. 150 newtons
9. 1.5
10. Answers are:
    a. 1,500 newtons
    b. 2 meters

4.2 Mechanical Advantage of Simple Machines

1. 5
2. 1.5
3. 0.5 meters
4. 4.8 meters
5. 0.4
6. 0.8 meters
7. 0.25 meters
8. 6.7
9. 2 meters
10. 12 meters
11. 2.4
12. 6 newtons
13. 560 newtons
14. 4 meters

4.2 Gear Ratios

1. 9 turns
2. 1 turn
3. 4 turns
4. 10 turns
5. 6 turns
6. Answers for the table are:

Table 1: Using the gear ratio to calculate number of turns

<table>
<thead>
<tr>
<th>Input Gear (# of teeth)</th>
<th>Output Gear (# of teeth)</th>
<th>Gear ratio (Input Gear: Output Gear)</th>
<th>How many turns does the output gear make if the input gear turns 3 times?</th>
<th>How many turns does the input gear make if the output gear turns 2 times?</th>
</tr>
</thead>
<tbody>
<tr>
<td>24</td>
<td>24</td>
<td>1</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>36</td>
<td>12</td>
<td>3</td>
<td>9</td>
<td>0.67, or 2/3 of a turn</td>
</tr>
<tr>
<td>24</td>
<td>36</td>
<td>0.67, or 2/3</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>48</td>
<td>36</td>
<td>1.33, or 4/3</td>
<td>4</td>
<td>1.5</td>
</tr>
<tr>
<td>24</td>
<td>48</td>
<td>0.5, or 1/2</td>
<td>1.5</td>
<td>4</td>
</tr>
</tbody>
</table>

7. Answers for the table are:

Table 2: Set up for three gears

<table>
<thead>
<tr>
<th>Set up</th>
<th>Gears</th>
<th>Number of teeth</th>
<th>Ratio 1 (top gear: middle gear)</th>
<th>Ratio 2 (middle gear: bottom gear)</th>
<th>Total gear ratio (Ratio 1 x Ratio 2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Top gear</td>
<td>12</td>
<td>1</td>
<td>2</td>
<td>1/3</td>
</tr>
<tr>
<td></td>
<td>Middle gear</td>
<td>24</td>
<td></td>
<td>2/3</td>
<td>2/1</td>
</tr>
<tr>
<td></td>
<td>Bottom gear</td>
<td>36</td>
<td></td>
<td>3</td>
<td>3/1</td>
</tr>
<tr>
<td>2</td>
<td>Top gear</td>
<td>24</td>
<td>2/3</td>
<td>3</td>
<td>2/1</td>
</tr>
<tr>
<td></td>
<td>Middle gear</td>
<td>36</td>
<td></td>
<td>3</td>
<td>3/1</td>
</tr>
<tr>
<td></td>
<td>Bottom gear</td>
<td>12</td>
<td></td>
<td>1/2</td>
<td>1/3</td>
</tr>
</tbody>
</table>
Table 2: Set up for three gears

<table>
<thead>
<tr>
<th>Set up</th>
<th>Gears</th>
<th>Number of teeth</th>
<th>Ratio 1 (top gear: middle gear)</th>
<th>Ratio 2 (middle gear: bottom gear)</th>
<th>Total gear ratio (Ratio 1 x Ratio 2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>Top gear</td>
<td>12</td>
<td>$\frac{1}{4}$</td>
<td>$\frac{4}{2}$</td>
<td>$\frac{1}{2}$</td>
</tr>
<tr>
<td></td>
<td>Middle gear</td>
<td>48</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Bottom gear</td>
<td>24</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>Top gear</td>
<td>24</td>
<td>$\frac{1}{2}$</td>
<td>$\frac{4}{3}$</td>
<td>$\frac{2}{3}$</td>
</tr>
<tr>
<td></td>
<td>Middle gear</td>
<td>48</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Bottom gear</td>
<td>36</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

4.3 Efficiency

5. Answers are:
   a. 2,025 million watts
   b. 39.5 percent
6. 59 percent

5.2 Equilibrium

1. The answer is:

2. 142 N
3. A is 50 N; B is 50 N
4. A is 20 N, B is 20 N, and C is 8 N.
5. A force applied at right angles to the path of the asteroid will result in a new acceleration that may turn its path enough to avoid striking Earth. Any variation from a right angle force wastes the available energy by simply making an insignificant change in the asteroid’s velocity.

6. From the outside of a balloon, two forces act inward. The elastic membrane of the balloon and the pressure of Earth’s atmosphere work together to balance the outward force of the helium compressed inside. Together with the elastic force, atmospheric pressure near Earth’s surface applies enough force to maintain this equilibrium, but as the balloon rises, atmospheric pressure decreases. Although the inward force supplied by the elastic membrane remains unchanged, the decreasing atmospheric pressure force causes an imbalance with the outward force of the contained helium and the balloon expands. At some point, the membrane of the balloon reaches its elastic limit and bursts.
5.4 Torque

1. 117.6 N-m clockwise
2. Answers are:
   a. \( m_2 = 2 \) kilograms
   b. \( \frac{m_1}{m_2} = 3 \)
   c. It should be placed to the right. The torque due to \( m_1 \) is greater than the torque due to \( m_2 \).
   d. \( m_1 = 8 \) centimeters
3. Answers are:
   a. 20 N force: 3.0 N-m clockwise or –3.0 N-m
   b. 15 N force: 3.75 N-m counterclockwise or +3.75 N-m
   c. 5 N force: 1.75 N-m clockwise or –1.75 N-m
   d. 1.0 N-m clockwise or –1.0 N-m
   e. The force creates a counterclockwise torque of 1.0 N-m to balance the other torques.
   f. \( F = 3.33 \) N upward

6.1 Pythagorean Theorem

1. 5
2. 5
3. 5
4. 13.4 cm
5. Answers are:
   a. (3,3)
   b. 4.2

Challenge problems:
6. \( \pm 21.9 \) or \(-21.9\)

6.1 Adding Displacement Vectors

Practice set 1:
1. The total displacement is 5 meters east and 5 meters north.
2. The total displacement is 2 meters east and 2 meters south.
3. The total displacement is zero. Total distance traveled is 40 meters.

Practice set 2:
1. \( \vec{x_R} = (1, -5) \) m
2. \( \vec{x_R} = (3, 3) \) m; diagram at right:
3. \( \vec{x_R} = (5, 5) \) m
4. \( \vec{x_R} = (6, 2) \) m
5. \( \vec{x_R} = (2, 0) \) m

6.1 Projectile Motion

1. Answers are:
   a. horizontal and vertical distance
   b. horizontal speed
   c. \( d_x = v_x t; \quad d_y = 4.9t^2 \)
   d. 6.4 m/sec
2. Answers are:
   a. horizontal speed, time
   b. vertical distance, horizontal distance
   c. \( d_x = 4.9t^2; \quad d_y = v_y t \)
   d. height = 44.1 meters, horizontal distance = 30 meters
3. Answers are:
   a. vertical distance
   b. time
   c. \( d_y = 4.9t^2 \)
   d. 1.4 seconds
4. Answers are:
   a. horizontal and vertical distance
   b. horizontal speed
   c. \( d_x = v_x t; \quad d_y = 4.9t^2 \)
   d. 59 m/sec
5. Answers are:
   a. horizontal speed, time
   b. vertical distance
   c. \( d_y = 4.9t^2 \)
   d. 2.8 meters
6. Answers are:
   a. horizontal speed, vertical distance
   b. time
   c. \( d_y = 4.9t^2 \)
   d. 0.45 seconds
7. Answers are:
   a. height of bridge, time
   b. height of person
   c. bridge height - vertical distance marshmallow travels = person's height; $d_y = 4.9t^2$
   d. marshmallow travels 3.38 meters; person's height = 1.62 meters

6.2 Circular Motion

1. Answers are:
   a. 1,188°/second
   b. 200 rpm
2. Answers are:
   a. 0.38 meter
   b. 1.52 m/sec
   c. 0.75 m/sec
3. Slower. A cd rotates at about 500 rpm when the head reads the inner edge and 200 rpm when the head reads the outer edge.
4. Answers are:
   a. 0.96 meter
   b. 2.15 meter
   c. 1,042 rews
   d. 465 rews

6.3 Universal Gravitation

1. $F = 9.34 \times 10^{-6} \text{ N}$. This is basically the force between you and your car when you are at the door.
2. $5.28 \times 10^{-10} \text{ N}$
3. 4.42 N
4. $7.33 \times 10^{22} \text{ kilograms}$
5. Answers are:
   a. $9.8 \text{ N/kg} = 9.8 \text{ kg-m/sec}^2 \cdot \text{kg} = 9.8 \text{ m/sec}^2$
   b. Acceleration due to the force of gravity of Earth.
   c. Earth’s mass and radius.
6. $1.99 \times 10^{20} \text{ N}$
7. 4,848 N
8. $3.52 \times 10^{22} \text{ N}$

Unit 3 Practice Sheets

7.1 Indirect Measurement

1. The tree is 3 meters tall.
2. The flagpole’s shadow is 12.75 meters.
3. Answers are:
   a. 2,240 feet
   b. 672 meters
4. Each apple is 0.1 kilogram or 100 grams.
5. Each staple is 0.0324 gram or 3.24 milligrams.
6. One business card is 0.0336 centimeter thick.
7. The thickness of a CD is approximately 0.13 centimeter or 1.3 millimeters.
8. Answers are:
   a. 4.8 millimeters
   b. 9.6 millimeters
   c. 48 millimeters
   d. 9,600 millimeters or 9.6 meters
9. Answers are:
   a. Each cheesecake takes 0.85 hour or 51 minutes to make.
   b. Yvonne earns $12 per cheesecake.
   c. Yvonne earns $14.12 per hour.
10. The mass of the block of marble is 324,000 grams or 324 kilograms.
11. Sample answer:
    First, fill the dropper with water from the glass. Then, place drops of water one-by-one into the graduated cylinder. Count the number of drops it takes to reach the 5.0 mL mark on the graduated cylinder. To find the volume of one drop, divide the value 5.0 mL by the number of drops.
12. Sample answer:
    (1) Remove the newspaper from the recycling bin.
    (2) Unfold each sheet and smooth the paper.
    (3) Neatly stack the sheets of paper.
    (4) Place the newspaper on a flat surface.
    (5) Place something heavy, like a hardbound book, on the newspaper to remove excess space between the sheets of paper.
    (6) Measure the height of the stack of newspaper.
    (7) Divide the stack of paper by the number of sheets.
    Note to teacher: This question is designed to prompt students to think about sources of experimental error. You may wish to ask the students what would happen if they divided the height of the recycle bin by the number of sheets of newsprint multiplied by 4. Why wouldn’t this method yield an accurate result?

7.2 Temperature Scales

Practice set 1:
1. 7.2°C
2. 177°C
3. 107°C
4. 375°F
5. 450°F
6. The table shows that the friend in Denmark thinks that the temperature is on the Celsius scale because 15°C is equal to 59°F, a warm temperature. However, 15°F is a cold temperature, equivalent to -9.4°C.

<table>
<thead>
<tr>
<th>°F</th>
<th>°C</th>
</tr>
</thead>
<tbody>
<tr>
<td>15°F</td>
<td>-9.4°C</td>
</tr>
<tr>
<td>59°F</td>
<td>15°C</td>
</tr>
</tbody>
</table>
Practice set 2:
1. 98 K
2. 275 K = 2°C No, the mysterious, silver substance has a much higher melting point than mercury.

3. The thermometer is calibrated to the Fahrenheit scale. On the Kelvin scale, 90 K is too cold (-298°F and -183°C), and 90°C is too hot, just 10 degrees less than the boiling point of water (100°C or 212°F).

<table>
<thead>
<tr>
<th>°F</th>
<th>°C</th>
<th>K</th>
</tr>
</thead>
<tbody>
<tr>
<td>90°F</td>
<td>32°C</td>
<td>305 K</td>
</tr>
<tr>
<td>194°F</td>
<td>90°C</td>
<td>363 K</td>
</tr>
<tr>
<td>-298°F</td>
<td>-183°C</td>
<td>90 K</td>
</tr>
</tbody>
</table>

7.3 Specific Heat

1. 3,150 joules
2. 14,644 joules
3. 4,700 joules
4. 6,276,000 joules
5. 2,500 J/kg°C; wood
6. 800 J/kg°C; glass
7. 0.25 kilogram
8. 12 kilogram
9. 4,800 joules
10. Yes, it would take 669, 440 joules to heat 2 kilograms water by 80°C.
11. About 22°C
12. About 1°C
13. Gold would heat up the quickest because it has the lowest specific heat.
14. Pure water is the best insulator because it has the highest specific heat.
15. Steel is a better conductor of heat than wood because its specific heat is lower than that for wood.

8.1 Density

1. 1.10 g/cm³
2. 0.87 g/cm³
3. 2.70 g/cm³
4. 920,000 grams or 920 kilograms
5. 2,420 grams or 2.42 kilograms
6. 1,025 grams or 1.025 kilograms
7. 1,000 cm³
8. 29.8 cm³
9. 11.4 mL
10. Answers are:
    a. density = 960 kg/m³, HDPE
    b. 76,000 grams or 76 kilograms
    c. The volume needed is 0.11 m³; 11 10-liter containers would be needed to hold the plastic
    d. HDPE, LDPE, PP (PS would probably be suspended in seawater)

8.1 Stress

1. 15,000 N/m²
2. The wooden beam is made of pine which has a tensile strength of 60 MPa or 60,000,000 Pa.
3. 135 newtons of force could break three pencils.
4. 0.4 m²
5. 0.1 m²

8.2 Buoyancy

1. Sink
2. Float
3. 0.12 N
4. 0.10 N
5. The light corn syrup has greater buoyant force than the vegetable oil.
6. 0.13 N
7. The buoyant force would be smaller if the gold cube were suspended in water. Student explanations may vary. A simple observation, such as “The water is thinner than the molasses” is acceptable, as well as the more sophisticated “The displaced water would weigh less than the displaced molasses” or “The water is less dense than the molasses.”

8.2 Archimedes Principle

1. Answers are
   a. 0.069 N
   b. The paraffin will sink in the gasoline.
2. Answers are:
   a. 0.39 N
   b. The platinum will sink in the molasses.
3. Answers are:
   a. 1.95 N
   b. The gold will sink in the mercury.
4. Answers are:
   a. The density of paraffin is 0.87 g/cm³; the density of gasoline is 0.7 g/cm³.
   b. The density of platinum is 21.4 g/cm³; the density of molasses is 1.37 g/cm³.
   c. The density of gold is 19.3 g/cm³; the density of mercury is 13.6 g/cm³.
5. In each case, a material sinks in a fluid if it is more dense than the fluid. A material floats in a fluid if it is less dense than the fluid.
6. Answers are:
   a. Floats
   b. Floats
   c. Sinks
   d. Floats
   e. Sinks
8.3 Boyle’s Law

1. 3.25 atm
2. 36 m³
3. 563 kPa
4. 570 liters
5. 25 liters

8.3 Pressure-Temperature Relationship

1. 0.27 atmospheres
2. 1,000 K
3. Answers are:

<table>
<thead>
<tr>
<th>P₁</th>
<th>T₁</th>
<th>P₂</th>
<th>T₂</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. 30.0 atm</td>
<td>-100°C (173 K)</td>
<td>134 atm</td>
<td>500°C (773 K)</td>
</tr>
<tr>
<td>b. 15.0 atm</td>
<td>25.0°C (298 K)</td>
<td>18.0 atm</td>
<td>87.0°C (360 K)</td>
</tr>
<tr>
<td>c. 5.00 atm</td>
<td>490 K</td>
<td>3.00 atm</td>
<td>293 K</td>
</tr>
</tbody>
</table>

8.3 Charles’ Law

1. 25.4 liters
2. 22.8 liters
3. Answers are:

<table>
<thead>
<tr>
<th>V₁</th>
<th>T₁</th>
<th>V₂</th>
<th>T₂</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. 6,114 mL</td>
<td>838 K</td>
<td>1,070 mL</td>
<td>147 K</td>
</tr>
<tr>
<td>b. 3,250 mL</td>
<td>475°C (748 K)</td>
<td>1,403 mL</td>
<td>50°C (323 K)</td>
</tr>
<tr>
<td>c. 10 L</td>
<td>-56°C (217 K)</td>
<td>15 L</td>
<td>50°C (323 K)</td>
</tr>
</tbody>
</table>

9.1 The Structure of the Atom

1. Answers are:

<table>
<thead>
<tr>
<th>What is this element?</th>
<th>How many electrons does the neutral atom have?</th>
<th>What is the mass number?</th>
</tr>
</thead>
<tbody>
<tr>
<td>lithium</td>
<td>3</td>
<td>7</td>
</tr>
<tr>
<td>carbon</td>
<td>6</td>
<td>12</td>
</tr>
<tr>
<td>hydrogen</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>hydrogen (a radioactive isotope, 3H, called tritium)</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>beryllium</td>
<td>4</td>
<td>9</td>
</tr>
</tbody>
</table>

2. The atomic mass of 1.00794 amu represents an average of the masses and abundance of all to the hydrogen isotopes (hydrogen-1, hydrogen-2, and hydrogen-3). Because this atomic mass is so close to "1," we can assume that the most abundant isotope is hydrogen-1.

3. Answers are:
   a. hydrogen-2: 1 proton, 1 neutron
   b. scandium-45: 21 protons, 24 neutrons
   c. aluminum-27: 13 protons, 14 neutrons
   d. uranium-235: 92 protons, 143 neutrons
   e. carbon-12: 6 protons, 6 neutrons
4. Most of an atom’s mass is concentrated in the nucleus. The number of electrons and protons is the same but electrons are so light they contribute very little mass. The mass of the proton is 1,835 times the mass of the electron. Neutrons have a bit more mass than protons, but the two are so close in size that we usually assume their masses are the same.
5. No, it has a proton (+1) and no electrons to balance charge. Therefore, the overall charge of this atom (now, called an ion) is +1.
6. This sodium atom has 10 electrons, 11 protons, and 12 neutrons.
9.2 Dot Diagrams

<table>
<thead>
<tr>
<th>Element</th>
<th>Chemical Symbol</th>
<th>Total Electrons</th>
<th>No. of Valence Electrons</th>
<th>Dot Diagram</th>
</tr>
</thead>
<tbody>
<tr>
<td>Potassium</td>
<td>K</td>
<td>19</td>
<td>1</td>
<td>K·</td>
</tr>
<tr>
<td>Nitrogen</td>
<td>N</td>
<td>7</td>
<td>5</td>
<td>N·</td>
</tr>
<tr>
<td>Carbon</td>
<td>C</td>
<td>6</td>
<td>4</td>
<td>C·</td>
</tr>
<tr>
<td>Beryllium</td>
<td>Be</td>
<td>4</td>
<td>2</td>
<td>Be·</td>
</tr>
<tr>
<td>Neon</td>
<td>Ne</td>
<td>10</td>
<td>8</td>
<td>Ne·</td>
</tr>
<tr>
<td>Sulfur</td>
<td>S</td>
<td>16</td>
<td>6</td>
<td>S·</td>
</tr>
</tbody>
</table>

Unit 4 Skill and Practice Sheets

10.2 Power in Flowing Energy

1. Answers are given in table below:

<table>
<thead>
<tr>
<th>Force (N)</th>
<th>Distance (m)</th>
<th>Time (sec)</th>
<th>Work (J)</th>
<th>Power (W)</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>2</td>
<td>5</td>
<td>200</td>
<td>40</td>
</tr>
<tr>
<td>100</td>
<td>2</td>
<td>10</td>
<td>200</td>
<td>20</td>
</tr>
<tr>
<td>100</td>
<td>4</td>
<td>10</td>
<td>400</td>
<td>40</td>
</tr>
<tr>
<td>100</td>
<td>5</td>
<td>25</td>
<td>500</td>
<td>20</td>
</tr>
<tr>
<td>50</td>
<td>20</td>
<td>20</td>
<td>1000</td>
<td>50</td>
</tr>
<tr>
<td>20</td>
<td>30</td>
<td>10</td>
<td>600</td>
<td>60</td>
</tr>
<tr>
<td>9</td>
<td>20</td>
<td>3</td>
<td>180</td>
<td>60</td>
</tr>
<tr>
<td>3</td>
<td>25</td>
<td>15</td>
<td>75</td>
<td>5</td>
</tr>
</tbody>
</table>

2. Answers are:
   a. 1,800 J
   b. 450 watts

3. Answers are:
   a. 60,000 J
   b. 2,000 W; 2.68 hp

4. Answers are:
   a. 36,750 J (37,000 J with correct significant figures)
   b. 20.4 W (20 W with correct significant figures)

5. 200 W

6. Answers are:
   a. 5,000 seconds or 1.4 hours
   b. 8,640,000 J
   c. 17.3 apples

7. Answers are:
   a. 98,000 J
   b. 98,000 W; 131 hp
   c. The first hill is the tallest because a roller coaster loses energy as it moves along the track. No roller coaster is 100% efficient. Unless there is a motor to give it additional energy, it will never be able to make it back up to a height as high as the first hill.

8. 3.33 W

10.2 Efficiency and Energy

1. 55%
2. 12%
3. Answers are:
   a. 91%
   b. Energy is lost due to friction with the track (which creates heat), air resistance, and the sound made by the track and wheels.

11.2 Balancing Chemical Equations

<table>
<thead>
<tr>
<th>Reactants</th>
<th>Products</th>
<th>Chemical Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hydrochloric acid</td>
<td>Water H₂O</td>
<td>HCl + NaOH → NaCl + H₂O</td>
</tr>
<tr>
<td>and Sodium hydroxide</td>
<td>Sodium chloride</td>
<td></td>
</tr>
<tr>
<td>NaOH</td>
<td>NaCl</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Reactants</th>
<th>Products</th>
<th>Chemical Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Calcium carbonate</td>
<td>Potassium carbonate</td>
<td>CaCO₃ + KI → K₂CO₃ + CaI₂</td>
</tr>
<tr>
<td>and Potassium iodide</td>
<td>and Calcium iodide</td>
<td></td>
</tr>
<tr>
<td>KI</td>
<td>Cal₂</td>
<td></td>
</tr>
</tbody>
</table>
Practice set 2:
1. $4\text{Al} + 3\text{O}_2 \rightarrow 2\text{Al}_2\text{O}_3$
2. $\text{CO} + 3\text{H}_2 \rightarrow \text{H}_2\text{O} + \text{CH}_4$
3. $2\text{HgO} \rightarrow 2\text{Hg} + \text{O}_2$
4. $\text{CaCO}_3 \rightarrow \text{CaO} + \text{CO}_2$ (already balanced)
5. $3\text{Fe} + 2\text{Fe}_2\text{O}_3 \rightarrow 4\text{Fe} + 3\text{CO}_2$
6. $\text{N}_2 + 3\text{H}_2 \rightarrow 2\text{NH}_3$
7. $2\text{K} + 2\text{H}_2\text{O} \rightarrow 2\text{KOH} + \text{H}_2$
8. $4\text{P} + 5\text{O}_2 \rightarrow 2\text{P}_2\text{O}_5$
9. $\text{Ba(OH)}_2 + \text{H}_2\text{SO}_4 \rightarrow 2\text{H}_2\text{O} + \text{BaSO}_4$
10. $\text{CaF}_2 + \text{H}_2\text{SO}_4 \rightarrow \text{CaSO}_4 + 2\text{HF}$
11. $3\text{C} + 2\text{Fe}_2\text{O}_3 \rightarrow 4\text{Fe} + 3\text{CO}_2$

11.3 Radioactivity

1. In the answers below, “a” is alpha decay and “b” is beta decay.

a. Answers are:
   $^{238}\text{U} \rightarrow ^{234}\text{Th}$
   $^{226}\text{Ra} \rightarrow ^{222}\text{Rn}$
   $^{214}\text{Po} \rightarrow ^{210}\text{Pb}$

b. Answers are:
   $^{240}\text{Pu} \rightarrow ^{236}\text{Np}$
   $^{228}\text{Bi} \rightarrow ^{224}\text{Ra}$

2. Answers are:
   a. During 11 minutes, fluorine-18 would experience 6 half-lives.
   b. 0.16 gram would be left after 11 minutes (660 seconds).

3. The amount after 28,650 years would be $0.0313m$ where $m$ is the mass of the sample.

4. For one-fourth of the original mass to be left, there must have been time for two half-lives. Therefore, the half-life for this radioactive isotope is 9 months.

5. Answers are:
   a. $0.8\ \text{W/m}^2$
   b. $3.6 \times 10^{13}$ reactions per second

12.1 Einstein’s Formula

1. $9 \times 10^{13}\ \text{J}$
2. $5.4 \times 10^{18}\ \text{J}$
3. $9 \times 10^{12}\ \text{J}$
4. Answers are:
   a. $3.87 \times 10^{26}\ \text{J}$
   b. $3.87 \times 10^{26}\ \text{W}$
   c. $1.36 \times 10^{17}\ \text{kg}$

5. Answers are:
   a. $4,444\ \text{kg}$
   b. Yes, we get much more energy from the sun than we use in one year.

Unit 5 Skill and Practice Sheets

13.2 Using an Electrical Meter

1. Sample diagram:

2. First battery: 1.553 volts; second battery: 1.557 volts
3. First bulb: 1.514 volts; second bulb: 1.586 volts
4. 3.113 volts
5. 3.108 volts
6. The two voltages are approximately equal.
7. post #1: 0.0980 amps
   post #2: 0.0981 amps
8. The current is approximately the same at all points.
9. First bulb: 15.4 ohms; second bulb: 16.2 ohms
10. Measuring resistance: First, set the meter dial to measure resistance. Remove the bulb from its holder. Then place one lead on the side of the metal portion of the light bulb (where the bulb is threaded to fit into the socket). Place the other lead on the “bump” at the base of the light bulb. The meter will display the bulb’s resistance.
11. Measuring voltage: First, set the meter dial to measure DC voltage. Locate the device (battery, bulb, etc.) that you wish to measure the voltage across. Then place one meter lead on one of the posts next to the device. Put the other meter lead on the post on the other side of the device. The meter will display the device’s voltage. If it shows a negative voltage, switch the two leads.
12. Measuring current: First, set the meter to measure DC current.
Then break the circuit at the location where you wish to measure the current. Connect one of the meter leads at one side of the break. Connect the other lead at the other side of the break. The meter will display the current. If it shows a negative current, switch the two leads.

### 13.3 Ohm’s Law

1. 3 amps
2. 0.75 amp
3. 0.5 amp
4. 1 amp
5. 120 volts
6. 8 volts
7. 50 volts
8. 12 ohms
9. 240 ohms
10. 1.5 ohms
11. 3 ohms
12. Answers are:
   a. Circuit A: 6 V; Circuit B: 12 V
   b. Circuit A: 1 A; Circuit B: 2 A
   c. Circuit A: 0.5 A; Circuit B: 1 A
   d. It is brighter in circuit B because there is a greater voltage and greater current (and more power is consumed since power equals current times voltage).

### 13.4 Series Circuits

1. Answers are:
   a. 6 volts
   b. 2 ohms
   c. 3 amps
   d. 3 volts
   e. Diagram:

2. Answers are:
   a. 6 volts
   b. 3 ohms
   c. 2 amps
   d. 2 volts
   e. Diagram:

3. The current decreases because the resistance increases.
4. The brightness decreases because the voltage across each bulb decreases and the current decreases. Since power equals voltage times current, the power consumed also decreases.
5. Answers are:
   a. 3 ohms
   b. 2 amps
   c. 1 ohm bulb: 2 volts; 2 ohm bulb: 4 volts
6. Answers are:
   a. 12 volts
   b. 4 ohms
   c. 3 amps
   d. 6 volts
   e. Diagram:

7. Answers are:
   a. 2 ohms
   b. 1 volt
   c. Diagram:
8. Answers are:
a. 6 ohms
b. 1.5 amps
c. 2 ohm resistor: 3 volts; 3 ohm resistor: 4.5 volts; 1 ohm resistor: 1.5 volt
d. The sum is 9 volts, the same as the battery voltage.

9. Answers are:
a. Diagram A: 0.5 amps; Diagram B: 1.0 amps
b. Diagram A: 0.25 amps; Diagram B: 0.5 amps
c. The amount of current increases.

14.2 Parallel Circuits

Practice set 1:
1. Answers are:
   a. 12 volts
   b. 6 amps
   c. 12 amps
   d. 1 ohm
2. Answers are:
   a. 12 volts
   b. 4 amps
   c. 8 amps
   d. 1.5 ohms
3. Answers are:
   a. 12 volts
   b. 2 ohm branch: 6 amps; 3 ohm branch: 4 amps
   c. 10 amps
   d. 1.2 ohms

14.3 Electrical Power

1. Answers are:
   a. 5 kW
   b. 10 kWh
   c. $1.50
2. Answers are:
   a. 300 minutes
   b. 5 hours
   c. 1.2 kW
   d. 6 kW
   e. $0.90
3. 960 W
4. 24 W
5. Answers are:
   a. 60 W
   b. 0.06 kW
   c. 525.6 kWh
   d. $78.84
6. 0.625 A
7. Answers are:
   a. 3 V
   b. 1 A
   c. 3 W

8. Answers are:
   a. 24 ohms
   b. 600 W
   c. 0.6 kW
9. Answers are:
   a. 20.5 A
   b. 10.8 ohms
   c. 18 kWh
   d. $140.40
10. Answers are:
    a. 6 ohms
    b. 2 A
    c. 12 W
    d. 24 W
11. Answers are:
    a. 12 V
    b. 4 A
    c. 48 W
    d. 8 A
    e. 96 W

15.2 Coulomb's Law

Practice set 1:
1. The force becomes \(\frac{1}{9}\) as strong.
2. The force becomes \(\frac{1}{16}\) as strong.
3. The force quadruples.
4. The force doubles.
5. The force quadruples.
6. The force does not change.
7. The forces becomes 16 times as large.

Practice set 2:
1. \(9 \times 10^9\) N
2. \(2.16 \times 10^9\) N
3. 3375 N
4. \(3.38 \times 10^6\) N
5. 5.63 N
6. 0.00556 C
7. \(3.33 \times 10^{-4}\) C
8. 6.7 m
9. 0.03 m
10. \(2.96 \times 10^{-12}\) C
Unit 6 Skill and Practice Sheets

16.3 Magnetic Earth

1. Scientists believe that the motion of molten metals in Earth’s outer core create its magnetic field.
2. Seven percent of 0.5 gauss is 0.035 gauss. In 100 years, Earth’s strength could be 0.465 gauss.
3. The poles could reverse within the next 2,000 years. During a reversal, the field would not completely disappear. The main magnetic field that we use for navigation would be replaced by several smaller fields with poles in different locations.
4. Rock provides a good record because as the rock is made, atoms in the rock align with the magnetic field of Earth. (Actually, oceanic rock is made of a substance called magnetite!) Rock that was made 750,000 years ago would have a north-south orientation that is exactly opposite the north-south orientation of rock that is made today. Therefore, we can use the north-south orientation of bands of rock on the sea floor to understand how many times the poles have reversed over geologic time.
5. NOTE: In many references, magnetic south pole is referred to as “magnetic north pole” because it is located at the geographic north pole. This terminology can be confusing to students who know that opposite poles attract. The north pole of a compass needle is in fact the north end of a bar magnet. This is why we think it is best to use the term magnetic south pole as the point to which the north end of a compass needle is attracted. For more information about Earth’s magnetism see http://www.ngdc.noaa.gov/.
   Answers are:
   a. Both the magnetic south pole and geographic north are located at the Arctic.
   b. The magnetic south pole is about 1,000 kilometers from the geographic north pole.

6. A compass needle is a bar magnet. It’s north pole is attracted to Earth’s magnetic south pole (if you consider the interior of Earth is like a bar magnet). Using magnetism, we can find our way using north as a reference point. However, using magnetism actually points us a bit off course because the magnetic south pole is not located at the same position as true north.
7. Answers are:
   a. Example problem: northeast.
   b. south
   c. west
   d. east
   e. southeast
   f. northwest
8. If you didn’t correct your compass for magnetic declination, you would be off course and possibly get lost.
9. Yes, magnetic declination equals zero on Earth’s surface along a line that goes from New Orleans through the eastern edge of Minnesota up through Churchill, Canada. However, the location of the zero-degree line is always changing. For more information about Earth’s magnetic declination and magnetism, see http://www.ngdc.noaa.gov/.

17.3 Transformers

1. 25 turns
2. 230 volts
3. 220 volts
4. 100 volts
5. 440 turns
6. 131 turns

18.1 The Inverse Square Law

1. 0.25 W/m²
2. one-ninth
3. 24,204 km (four times the original distance)
4. 55.6 N
5. It is 9 times more intense 2 meters away.
6. It is 16 times more intense at 1 meter than at 4 meters away.

18.2 Calculating Gravitational Fields

1. 8.9 N/kg
2. 24.9 N/kg
3. 10.4 N/kg
4. 10.9 N/kg
5. $8.5 \times 10^{25}$ kg
6. $6.0 \times 10^{24}$ kg
7. 271 N/kg

18.3 Calculating Electric Fields and Forces

1. 2.0 N
2. 0.008 N
3. 1.2 N
4. 63 N/C
5. 5.0 N/C
6. 25 N/C
The answer is:

1. 0.05 sec
2. 0.005 sec
3. 0.1 Hz
4. 3.33 Hz
5. period = 2 sec; frequency = 0.5 Hz
6. period = 0.25 sec; frequency = 4 Hz
7. period = 0.5 sec; frequency = 2 Hz

Answers are:

a. 5 sec
b. 5 sec
c. 0.2 Hz

Answers are:

a. 120 vibrations
b. 2 vibrations
c. 0.5 sec
d. 2 Hz

1. Answers are:
   a. A = 5 degrees; B = 100 cm
   b. A = 1 second; B = 2 seconds
2. Answers are:
   a. Diagram:

   
   [Diagram of harmonic motion graphs]

   20.1 Waves

1. Diagram:

   [Diagram of wave with labels for wavelength, crest, trough, amplitude, and half a wavelength]

   a. Two wavelengths
   b. The amplitude of a wave is the distance that the wave moves beyond the average point of its motion. In the graphic, the amplitude of the wave is 5 centimeters.
2. Answers are:
   a. Diagram:

   ![Diagram 1](image1)

   b. Diagram:

   ![Diagram 2](image2)

   3. 10 m/sec
   4. 5 m
   5. 10 Hz
   6. frequency = 0.5 Hz; speed = 2 m/sec
   7. frequency = 165 Hz; period = 0.006 sec
   8. A’s speed is 75 m/sec, and B’s speed is 65 m/sec, so A is faster.

9. Answers are:
   a. 4 sec
   b. 0.25 Hz
   c. 0.75 m/sec

20.1 Standing Waves

1. Answers are:
   a. \(A = 2\text{nd}; B = 3\text{rd}; C = 1\text{st or fundamental}; D = 4\text{th}\) (You can easily determine the harmonics of a vibrating string by counting the number of “bumps” on the string. The first harmonic (the fundamental) has one bump. The second harmonic has two bumps and so on.)
   b. Diagram:

   ![Diagram 3](image3)

   c. \(A = 1\) wavelength, \(B = 1.5\) wavelengths, \(C = \frac{1}{2}\) wavelength, \(D = 2\) wavelengths
   d. \(A = 3\) meters, \(B = 10\) meters, \(C = 6\) meters, \(D = 7.5\) meters

2. Answers are:
   a. Diagram:

   ![Diagram 4](image4)

   b. See answer for (c).

   c. See table below:

<table>
<thead>
<tr>
<th>Harmonic</th>
<th>Speed (m/sec)</th>
<th>Wavelength (m)</th>
<th>Frequency (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>36</td>
<td>24</td>
<td>1.5</td>
</tr>
<tr>
<td>2</td>
<td>36</td>
<td>12</td>
<td>3</td>
</tr>
<tr>
<td>3</td>
<td>36</td>
<td>8</td>
<td>4.5</td>
</tr>
<tr>
<td>4</td>
<td>36</td>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td>5</td>
<td>36</td>
<td>4.8</td>
<td>7.5</td>
</tr>
<tr>
<td>6</td>
<td>36</td>
<td>4</td>
<td>9</td>
</tr>
</tbody>
</table>

d. The frequency decreases as the wavelength increases. They are inversely proportional.
e. See table below.

<table>
<thead>
<tr>
<th>Harmonic</th>
<th>Speed (m/sec)</th>
<th>Wavelength (m)</th>
<th>Frequency (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>36</td>
<td>12</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>36</td>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td>3</td>
<td>36</td>
<td>4</td>
<td>9</td>
</tr>
<tr>
<td>4</td>
<td>36</td>
<td>3</td>
<td>12</td>
</tr>
<tr>
<td>5</td>
<td>36</td>
<td>2.4</td>
<td>15</td>
</tr>
<tr>
<td>6</td>
<td>36</td>
<td>2</td>
<td>18</td>
</tr>
</tbody>
</table>
f. The shorter rope resulted in harmonics with shorter wavelengths. The second harmonic on the short rope is equivalent in terms of wavelength and frequency to the 4th harmonic on the longer rope.
20.3 Wave Interference

1. Diagram:

2. 4 blocks
3. 2 blocks
4. 32 blocks
5. 4 blocks
6. 1 wavelength
7. 8 wavelengths
8. A portion of the table and a graphic of the new wave are shown below. The values for the third column of the table are found by adding the heights for wave 1 and wave 2.

<table>
<thead>
<tr>
<th>x (blocks)</th>
<th>Height wave 1 (blocks)</th>
<th>Height wave 2 (blocks)</th>
<th>Height of wave 1 +2 (blocks)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>0.8</td>
<td>2</td>
<td>2.8</td>
</tr>
<tr>
<td>2</td>
<td>1.5</td>
<td>0</td>
<td>1.5</td>
</tr>
<tr>
<td>3</td>
<td>2.2</td>
<td>-2</td>
<td>0.2</td>
</tr>
<tr>
<td>4</td>
<td>2.8</td>
<td>0</td>
<td>2.8</td>
</tr>
<tr>
<td>5</td>
<td>3.3</td>
<td>2</td>
<td>5.3</td>
</tr>
<tr>
<td>6</td>
<td>3.7</td>
<td>0</td>
<td>3.7</td>
</tr>
<tr>
<td>7</td>
<td>3.9</td>
<td>-2</td>
<td>-1.9</td>
</tr>
<tr>
<td>8</td>
<td>4</td>
<td>0</td>
<td>4</td>
</tr>
</tbody>
</table>

Question 8 (con’t)

9. The new wave looks like the second wave, but it vibrates about the position of the first wave, rather than about the zero line.

21.1 Decibel Scale

1. Twice as loud.
2. 55 dB
3. Answers are:
   a. 80 dB
   b. 60 dB
4. Four times louder
5. Answers are:
   a. 30 dB
   b. 50 dB
   c. 70 dB

Unit 8 Skill and Practice Sheets

22.1 Light Intensity Problems

1. Example problem: 4.8 W/m²
2. 0.0478 W/m²
3. 0.0119 W/m²
4. If distance from a light source doubles, then light intensity decreases by a factor of 4. Example: 4 × 0.0119 W/m² approximately equals 0.0478 W/m² (see questions 2 and 3).
5. Answers are:
   a. 0.005 W/m²
   b. 0.05 W/m²
   c. 0.5 W/m²
   d. 5 W/m²
6. The watts of a light source and light intensity are directly related. This means that if you use a light source that has 10 times the wattage, then light intensity will increase 10 times.

23.1 The Law of Reflection

1. Diagram at right:
2. The angle of reflection will be 20 degrees.
3. Each angle will measure 45 degrees.
4. Diagram at right:

5. The angle is 96 degrees. Therefore for the angles of incidence and reflection will each be 48 degrees.

6. The angles of incidence and reflection at point A are each 68.5 degrees; the angles of incidence and reflection at point B are each 21 degrees.

23.3 Refraction

Practice set 1:
1. The index of refraction will never be less than one because that would require the speed of light in a material to be faster than the speed of light in a vacuum. Nothing in the universe travels faster than that.
2. The index of refraction for air is less than that of glass because a gas like air is so much less dense than a solid like glass. The light rays are slowed each time they bump into an atom or molecule because they are absorbed and re-emitted by the particle. A light ray in a solid bumps into many more particles than a light ray traveling through a gas.
3. water: $2.26 \times 10^8$; glass: $2.0 \times 10^8$; diamond: $1.24 \times 10^8$
4. speed up

Practice set 2:
6. The light ray is moving from low-$n$ to high-$n$ so it will bend toward the normal.
7. The light ray is moving from high-$n$ to low-$n$ so it will bend away from the normal.
8. The difference in $n$ from diamond to water is 1.09 while the difference from sapphire to air is 0.770. The ray traveling from diamond to water experiences the greater change in $n$ so it would bend more.
9. From left to right, material B is water, emerald, helium, cubic zirconia.

23.3 Ray Diagrams

Practice set 1:
1. A is the correct answer. Light travels in straight lines and reflects off objects in all directions. This is why you can see something from different angles.
2. C is the correct answer. In this diagram, when light goes from air to glass it bends about 13 degrees from the path of the light ray in air. The light bends toward the normal to the air-glass surface because air has a lower index of refraction compared to glass. When the light re-enters the air, it bends about 13 degrees away for the light path in the glass and away from the normal.

As a ray of light approaches glass at an angle, it bends (refracts) toward the normal. As it leaves the glass, it bends away from the normal. However, if a ray of light enters a piece of glass perpendicular to the glass surface, the light ray will slow, but not bend because it is already in line with the normal. This happens because the index of refraction for air is lower than the index of refraction for glass. The index of refraction is a ratio that tells you how much light is slowed when it passes through a certain material.

3. A is the correct answer. Light rays that approach the lens that are in line with a normal to the surface pass right through, slowing but not bending. This is what happens at the principal axis. However, due to the curvature of the lens, the parallel light rays above and below the principal axis, hit the lens surface at an angle. These rays bend toward the normal (this bending occurs toward the fat part of the lens) and are focused at the lens’ focal point. The rays cross past the focal point.

4. Diagram:

Practice 2:
1. Diagram:

2. Diagram:

3. A lens acts like a magnifying glass if an object is placed to the left of a converging lens at a distance less than the focal length. The lens bends the rays so that they appear to be coming from a larger, more distant object than the real object. These rays you see form a virtual image. The image is virtual because the rays appear to come from an image, but don’t actually meet.
24.1 The Electromagnetic Spectrum

1. Green
2. Green
3. \(400 \times 10^{-9} \text{ m}\)
4. \(517 \times 10^{12} \text{ Hz}\)
5. \(652 \times 10^{-9} \text{ m}\)
6. \(566 \times 10^{12} \text{ Hz}\)
7. The answer is: \(\frac{\lambda_1}{\lambda_2} = \frac{f_2}{f_1}\)
8. \(\lambda = 0.122 \text{ meter}\)
9. \(\lambda = 3.3 \text{ meters}\)

10. \(f = 6 \times 10^{16} \text{ Hz}\)
11. \(1.0 \times 10^{15} \text{ Hz}\)
12. \(1.0 \times 10^{-4} \text{ Hz}\)
13. Answers are:
   a. \(f = 3 \times 10^{19} \text{ hertz}\)
   b. It is the minimum frequency.
14. radio waves, microwaves, infrared, visible, ultraviolet, X rays, and gamma rays
15. gamma rays, X rays, ultraviolet, visible, infrared, microwaves, and radio waves

24.1 Doppler Shift

1. Answers are:
   a. 20 nm
   b. \(5.6 \times 10^6 \text{ m/sec}\)
   c. The star is moving away from Earth.
2. Answers are:
   a. 16 nm
   b. \(8.6 \times 10^6 \text{ m/sec}\)
   c. The star is moving toward Earth.
3. Galaxy B is moving fastest because it has shifted farther toward the red (15 nm) than Galaxy A (9 nm).
4. It supports the Big Bang. This theory states that the universe began from a single point and has been expanding ever since.
5. Diagram at right:
6. Answers are:
   a. B and D
   b. A and C
   c. C
   d. B
7. Answers are:
   a. 10.5 nm
   b. longer
   c. 460.5 nm